Re-Imagined Time

Math – Trigonometry

Days 1 to 9

Instructions – Read ALL instructions carefully.

1. Please do one section per day. (front and back of page)
2. You will be asked to turn in the completed pages upon return to school.
3. If you need me you can contact me at
    a. vbale@k12.wv.us
    b. Through the live grades message board

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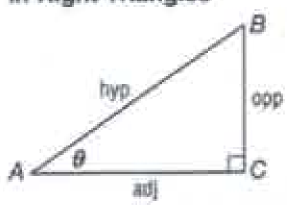
13-1 Study Guide and Intervention

Trigonometric Functions in Right Triangles

Trigonometric Functions for Acute Angles **Trigonometry** is the study of relationships among the angles and sides of a right triangle. A **trigonometric function** has a rule given by a **trigonometric ratio**, which is a ratio that compares the side lengths of a right triangle.

Trigonometric Functions in Right Triangles	If θ is the measure of an acute angle of a right triangle, *opp* is the measure of the leg opposite θ, *adj* is the measure of the leg adjacent to θ, and *hyp* is the measure of the hypotenuse, then the following are true.
	$\sin \theta = \frac{\text{opp}}{\text{hyp}}$ $\quad \cos \theta = \frac{\text{adj}}{\text{hyp}}$ $\quad \tan \theta = \frac{\text{opp}}{\text{adj}}$ $\csc \theta = \frac{\text{hyp}}{\text{opp}}$ $\quad \sec \theta = \frac{\text{hyp}}{\text{adj}}$ $\quad \cot \theta = \frac{\text{adj}}{\text{opp}}$

Example **In a right triangle, $\angle B$ is acute and $\cos B = \frac{3}{7}$. Find the value of $\tan B$.**

Step 1 Draw a right triangle and label one acute angle B. Label the adjacent side 3 and the hypotenuse 7.

Step 2 Use the Pythagorean Theorem to find b.

$a^2 + b^2 = c^2$	Pythagorean Theorem
$3^2 + b^2 = 7^2$	$a = 3$ and $c = 7$
$9 + b^2 = 49$	Simplify.
$b^2 = 40$	Subtract 9 from each side.
$b = \sqrt{40} = 2\sqrt{10}$	Take the positive square root of each side.

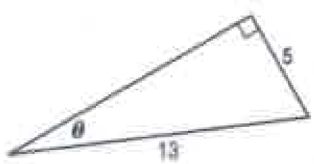
Step 3 Find $\tan B$.

$\tan B = \frac{\text{opp}}{\text{adj}}$	Tangent function
$\tan B = \frac{2\sqrt{10}}{3}$	Replace *opp* with $2\sqrt{10}$ and *adj* with 3.

Exercises

Find the values of the six trigonometric functions for angle θ.

1.



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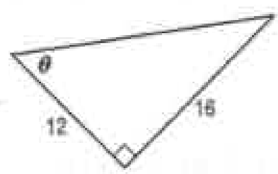

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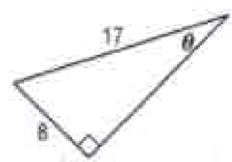

In a right triangle, $\angle A$ and $\angle B$ are acute.

4. If $\tan A = \frac{7}{12}$, what is $\cos A$?

5. If $\cos A = \frac{1}{2}$, what is $\tan A$?

6. If $\sin B = \frac{3}{8}$, what is $\tan B$?

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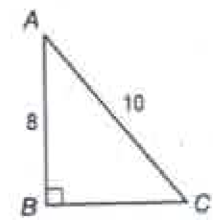
13-1 Study Guide and Intervention *(continued)*

Trigonometric Functions in Right Triangles

Use Trigonometric Functions You can use trigonometric functions to find missing side lengths and missing angle measures of right triangles. You can find the measure of the missing angle by using the inverse of sine, cosine, or tangent.

Example **Find the measure of $\angle C$. Round to the nearest tenth if necessary.**

You know the measure of the side opposite $\angle C$ and the measure of the hypotenuse. Use the sine function.



$\sin C = \frac{\text{opp}}{\text{hyp}}$	Sine function
$\sin C = \frac{8}{10}$	Replace *opp* with 8 and *hyp* with 10.
$\sin^{-1}\frac{8}{10} = m\angle C$	Inverse sine
$53.1° \approx m\angle C$	Use a calculator.

Exercises

Use a trigonometric function to find each value of x. Round to the nearest tenth if necessary.

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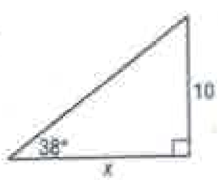

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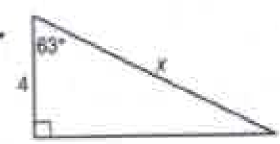

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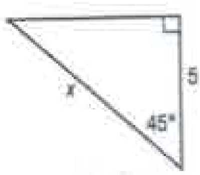

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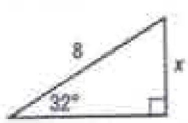
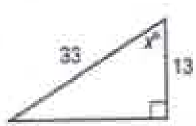


6.


Find x. Round to the nearest tenth if necessary.

7.


8.


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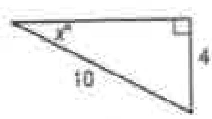

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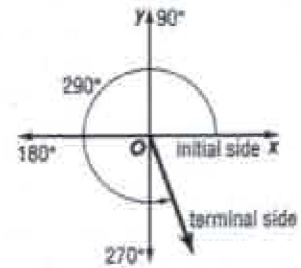
13-2 Study Guide and Intervention

Angles and Angle Measure

Angles in Standard Position An angle is determined by two rays. The degree measure of an angle in standard position is described by the amount and direction of rotation from the **initial side**, which lies along the positive x-axis, to the **terminal side**. A counterclockwise rotation is associated with positive angle measure and a clockwise rotation is associated with negative angle measure. Two or more angles in standard position with the same terminal side are called **coterminal angles**.

Example 1 **Draw an angle with measure 290° in standard position.**

The negative y-axis represents a positive rotation of 270°. To generate an angle of 290°, rotate the terminal side 20° more in the counterclockwise direction



Example 2 **Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.**

a. 250°

A positive angle is 250° + 360° or 610°. Add 360°.

A negative angle is 250° − 360° or −110°. Subtract 360°.

b. −140°

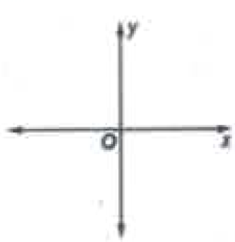
A positive angle is −140° + 360° or 220°. Add 360°.

A negative angle is −140° − 360° or −500°. Subtract 360°.

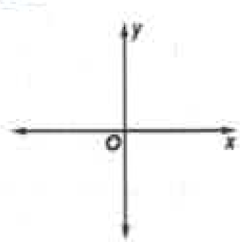
Exercises

Draw an angle with the given measure in standard position.

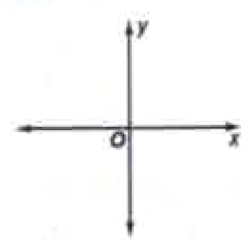
1. 160°



2. 280°



3. 400°



Find an angle with a positive measure and an angle with a negative measure that are coterminal with each angle.

4. 65° **5.** −75° **6.** 230° **7.** 420°

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13-2 Study Guide and Intervention *(continued)*

Angles and Angle Measure

Convert Between Degrees and Radians Angles can be measured in **degrees** and **radians**, which are units based on arc length. One radian is the measure of an angle θ in standard position with a terminal side that intercepts an arc with the same length as the radius of the circle. Degree measure and radian measure are related by the equations 2π radians = 360° and π radians = 180°.

Radian and Degree Measure	To rewrite the radian measure of an angle in degrees, multiply the number of radians by $\frac{180°}{\pi \text{ radians}}$. To rewrite the degree measure of an angle in radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180°}$.
Arc Length	For a circle with radius r and central angle θ(in radians), the arc length s equals the product of r and θ. $s = r\theta$

Example 1 **Rewrite each degree measure in radians and the radian measure in degrees.**

a. **45°**

$45° = 45°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{\pi}{4}$ radians

b. $\frac{5\pi}{3}$ **radians**

$\frac{5\pi}{3}$ radians $= \frac{5\pi}{3}\left(\frac{180°}{\pi}\right) = 300°$

Example 2 **A circle has a radius of 5 cm and central angle of 135°, what is the length of the circle's arc?**

Find the central angle in radians.

$135° = 135°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{3\pi}{4}$ radians

Use the radius and central angle to find the arc length.

$s = r\theta$	Write the formula for arc length.
$= 5 \cdot \frac{3\pi}{4}$	Replace r with 5 and θ with $\frac{3\pi}{4}$.
≈ 11.78	Use a calculator to simplify.

Exercises

Rewrite each degree measure in radians and each radian measure in degrees.

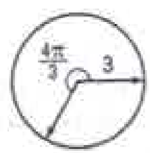
1. 140° **2.** −260° **3.** $-\frac{3\pi}{5}$

4. −75° **5.** $\frac{7\pi}{6}$ **6.** 380°

Find the length of each arc. Round to the nearest tenth.

7.


8.

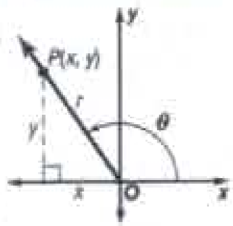

9.


13-3 Study Guide and Intervention

Trigonometric Functions of General Angles

Trigonometric Functions for General Angles

Trigonometric Functions, θ in Standard Position



Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ. By the Pythagorean Theorem, the distance r from the origin is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

$\sin \theta = \frac{y}{r}$ $\quad$ $\cos \theta = \frac{x}{r}$ $\quad$ $\tan \theta = \frac{y}{x}, x \neq 0$

$\csc \theta = \frac{r}{y}, y \neq 0$ $\quad$ $\sec \theta = \frac{r}{x}, x \neq 0$ $\quad$ $\cot \theta = \frac{x}{y}, y \neq 0$

Example **Find the exact values of the six trigonometric functions of θ if the terminal side of θ in standard position contains the point $(-5, 5\sqrt{2})$.**

You know that $x = -5$ and $y = 5$. You need to find r.

$r = \sqrt{x^2 + y^2}$ $\quad$ Pythagorean Theorem

$= \sqrt{(-5)^2 + (5\sqrt{2})^2}$ $\quad$ Replace x with -5 and y with $5\sqrt{2}$.

$= \sqrt{75}$ or $5\sqrt{3}$

Now use $x = -5$, $y = 5\sqrt{2}$, and $r = 5\sqrt{3}$ to write the six trigonometric ratios.

$\sin \theta = \frac{y}{r} = \frac{5\sqrt{2}}{5\sqrt{3}} = \frac{\sqrt{6}}{3}$ $\quad$ $\cos \theta = \frac{x}{r} = \frac{-5}{5\sqrt{3}} = -\frac{\sqrt{3}}{3}$ $\quad$ $\tan \theta = \frac{y}{x} = \frac{5\sqrt{2}}{-5} = -\sqrt{2}$

$\csc \theta = \frac{r}{y} = \frac{5\sqrt{3}}{5\sqrt{2}} = \frac{\sqrt{6}}{2}$ $\quad$ $\sec \theta = \frac{r}{x} = \frac{5\sqrt{3}}{-5} = -\sqrt{3}$ $\quad$ $\cot \theta = \frac{x}{y} = \frac{-5}{5\sqrt{2}} = -\frac{\sqrt{2}}{2}$

Exercises

The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

1. (8, 4)

2. (4, 4)

3. (0, 4)

4. (6, 2)

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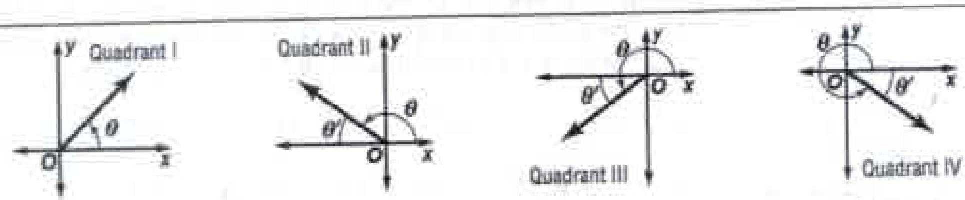
13-3 Study Guide and Intervention *(continued)*

Trigonometric Functions of General Angles

Trigonometric Functions with Reference Angles If θ is a nonquadrantal angle in standard position, its **reference angle** θ' is defined as the acute angle formed by the terminal side of θ and the x-axis.

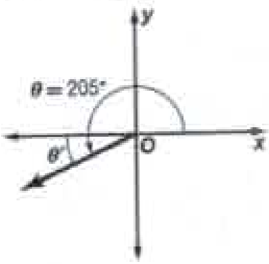
Reference Angle Rule



Quadrant I	Quadrant II	Quadrant III	Quadrant IV
$\theta' = \theta$	$\theta' = 180° - \theta$ $(\theta' = \pi - \theta)$	$\theta' = \theta - 180°$ $(\theta' = \theta - \pi)$	$\theta' = 360° - \theta$ $(\theta' = 2\pi - \theta)$

Example 1 **Sketch an angle of measure 205°. Then find its reference angle.**

Because the terminal side of 205° lies in Quadrant III, the reference angle θ' is $205° - 180°$ or $25°$.



Example 2 **Use a reference angle to find the exact value of $\cos \frac{3\pi}{4}$.**

Because the terminal side of $\frac{3\pi}{4}$ lies in Quadrant II, the reference angle θ' is $\pi - \frac{3\pi}{4}$ or $\frac{\pi}{4}$.

The cosine function is negative in Quadrant II.

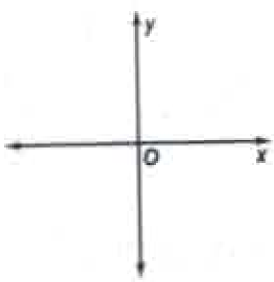
$\cos \frac{3\pi}{4} = -\cos \frac{\pi}{4} = -\frac{\sqrt{2}}{2}$.

Exercises

Sketch each angle. Then find its reference angle.

1. 155°

2. 230°



3. $\frac{4\pi}{3}$

4. $-\frac{\pi}{6}$

Find the exact value of each trigonometric function.

5. tan 330° **6.** $\cos \frac{11\pi}{4}$ **7.** cot 30° **8.** $\csc \frac{\pi}{4}$

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13-4 Study Guide and Intervention

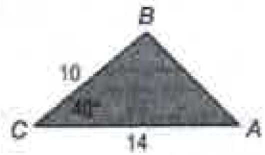
Law of Sines

Find the Area of a Triangle The area of any triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area of a Triangle	area $= \frac{1}{2}bc \sin A$ area $= \frac{1}{2}ac \sin B$ area $= \frac{1}{2}ab \sin C$	

Example **Find the area of $\triangle ABC$ to the nearest tenth.**

In $\triangle ABC$, $a = 10$, $b = 14$, and $C = 40°$.



$\text{Area} = \frac{1}{2}ab \sin C$ — Area formula

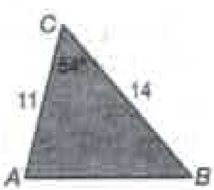
$= \frac{1}{2}(10)(14)\sin 40°$ — Substitution

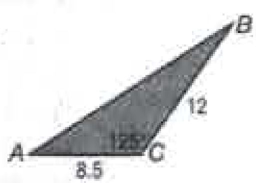
≈ 44.9951 — Simplify.

The area of the triangle is approximately 45 square units.

Exercises

Find the area of $\triangle ABC$ to the nearest tenth, if necessary.

1. 

2. 

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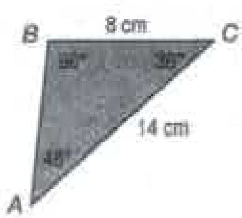
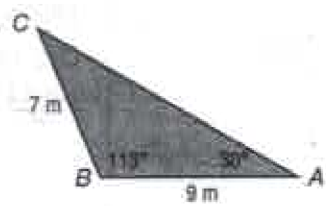
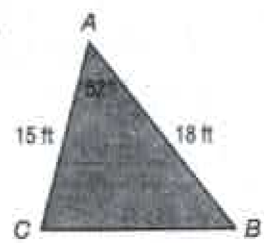
4. 

5. 

6. 

7. $A = 20°$, $c = 4$ cm, $b = 7$ cm

8. $C = 55°$, $a = 10$ m, $b = 15$ m

9. $B = 42°$, $c = 9$ ft, $a = 3$ ft

10. $c = 15$ in., $b = 13$ in., $A = 53°$

11. $a = 12$ cm, $b = 8$ cm, $C = 85°$

13-4 Study Guide and Intervention *(continued)*

Law of Sines

Use the Law of Sines to Solve Triangles You can use the Law of Sines to solve any triangle if you know the measures of two angles and any side opposite one of the angles, or the measures of two sides and the angle opposite one of them.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Possible Triangles Given Two Sides and One Opposite Angle	Suppose you are given a, b, and A for a triangle. If A is acute: $a < b \sin A \Rightarrow$ no solution $a = b \sin A \Rightarrow$ one solution $b > a > b \sin A \Rightarrow$ two solutions $a > b \Rightarrow$ one solution If A is right or obtuse: $a \leq b \Rightarrow$ no solution $a > b \Rightarrow$ one solution

Example **Determine whether $\triangle ABC$ has *no* solutions, *one* solution, or *two* solutions. Then solve $\triangle ABC$.**

a. $A = 48°$, $a = 11$, **and** $b = 16$ Since A is acute, find $b \sin A$ and compare it with a.
$b \sin A = 16 \sin 48° \approx 11.89$ Since $11 < 11.89$, there is no solution.

b. $A = 34°$, $a = 6$, $b = 8$
Since A is acute, find $b \sin A$ and compare it with a; $b \sin A = 8 \sin 34° \approx 4.47$. Since $8 > 6 > 4.47$, there are two solutions. Thus there are two possible triangles to solve.

Acute *B*

First use the Law of Sines to find B.

$$\frac{\sin B}{8} = \frac{\sin 34°}{6}$$

$\sin B \approx 0.7456$

$B \approx 48°$

The measure of angle C is about $180° - (34° + 48°)$ or about $98°$.

Use the Law of Sines again to find c.

$$\frac{\sin 98°}{c} \approx \frac{\sin 34°}{6}$$

$$c \approx \frac{6 \sin 98°}{\sin 34°}$$

$c \approx 10.6$

Obtuse *B*

To find B you need to find an obtuse angle whose sine is also 0.7456.

To do this, subtract the angle given by your calculator, $48°$, from $180°$. So B is approximately $132°$.

The measure of angle C is about $180° - (34° + 132°)$ or about $14°$.

Use the Law of Sines to find c.

$$\frac{\sin 14°}{c} \approx \frac{\sin 34°}{6}$$

$$c \approx \frac{6 \sin 14°}{\sin 34°}$$

$c \approx 2.6$

Exercises

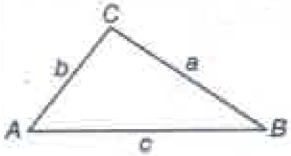
Determine whether each triangle has *no* solution, *one* solution, or *two* solutions. Then solve the triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1. $A = 50°$, $a = 34$, $b = 40$ **2.** $A = 24°$, $a = 3$, $b = 8$ **3.** $A = 125°$, $a = 22$, $b = 15$

13-5 Study Guide and Intervention

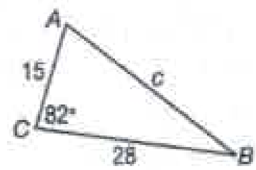
Law of Cosines

Use Law of Cosines to Solve Triangles

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides, and opposite angles with measures A, B, and C, respectively. Then the following equations are true.
	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

You can use the Law of Cosines to solve any triangle if you know the measures of two sides and the included angle (SAS case), or the measures of three sides (SSS case).

Example **Solve $\triangle ABC$.**



You are given the measures of two sides and the included angle. Begin by using the Law of Cosines to determine c.

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 28^2 + 15^2 - 2(28)(15)\cos 82°$

$c^2 \approx 892.09$

$c \approx 29.9$

Next you can use the Law of Sines to find the measure of angle A.

$\frac{\sin A}{a} = \frac{\sin C}{c}$

$\frac{\sin A}{28} \approx \frac{\sin 82°}{29.9}$

$\sin A \approx 0.9273$

$A \approx 68°$

The measure of B is about $180° - (82° + 68°)$ or about $30°$.

Exercises

Solve each triangle. Round side lengths to the nearest tenth and angle measures to the nearest degree.

1. $a = 14, c = 20, B = 38°$

2. $A = 60°, c = 17, b = 12$

3. $a = 4, b = 6, c = 3$

4. $A = 103°, b = 31, c = 52$

5. $a = 15, b = 26, C = 132°$

6. $a = 31, b = 52, c = 43$

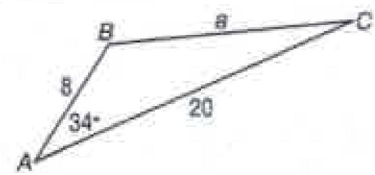
13-5 Study Guide and Intervention (continued)

Law of Cosines

Choose a Method to Solve Triangles

	Given	Begin by Using
Solving an Oblique Triangle	two angles and any side	Law of Sines
	two sides and an angle opposite one of them	Law of Sines
	two sides and their included angle	Law of Cosines
	three sides	Law of Cosines

Example **Determine whether $\triangle ABC$ should be solved by beginning with the Law of *Sines* or Law of *Cosines*. Then solve the triangle.**



You are given the measures of two sides and their included angle, so use the Law of Cosines.

$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$a^2 = 20^2 + 8^2 - 2(20)(8) \cos 34°$ $b = 20, c = 8, A = 34°$

$a^2 \approx 198.71$ Use a calculator to simplify.

$a \approx 14.1$ Use a calculator to simplify.

Use the Law of Sines to find C.

$\frac{\sin C}{c} = \frac{\sin A}{a}$ Law of Sines

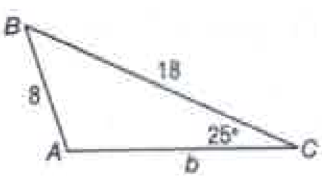
$\sin C \approx \frac{8 \sin 34°}{14.1}$ $c = 8, A = 34°, a = 14.1$

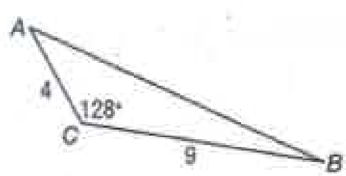
$C \approx 18°$ Use the $\sin^{-1}$ function.

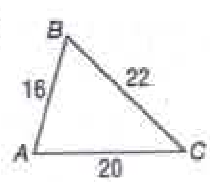
The measure of angle B is approximately $180° - (34° + 18°)$ or about $128°$.

Exercises

Determine whether each triangle should be solved by beginning with the Law of *Sines* or Law of *Cosines*. Then solve the triangle.

1. 

2. 

3. 

4. $A = 58°, a = 12, b = 8$

5. $a = 28, b = 35, c = 20$

6. $A = 82°, B = 44°, b = 11$

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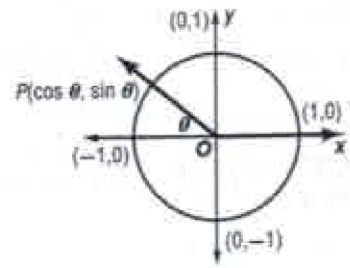
13-6 Study Guide and Intervention

Circular Functions

Circular Functions

Definition of Sine and Cosine	If the terminal side of an angle θ in standard position intersects the unit circle at $P(x, y)$, then $\cos \theta = x$ and $\sin \theta = y$. Therefore, the coordinates of P can be written as $P(\cos \theta, \sin \theta)$.	

Example **The terminal side of angle θ in standard position intersects the unit circle at $P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right)$. Find $\cos \theta$ and $\sin \theta$.**

$P\left(-\frac{5}{6}, \frac{\sqrt{11}}{6}\right) = P(\cos \theta, \sin \theta)$, so $\cos \theta = -\frac{5}{6}$ and $\sin \theta = \frac{\sqrt{11}}{6}$.

Exercises

The terminal side of angle θ in standard position intersects the unit circle at each point P. Find $\cos \theta$ and $\sin \theta$.

1. $P\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

2. $P(0, -1)$

3. $P\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$

4. $P\left(-\frac{4}{5}, -\frac{3}{5}\right)$

5. $P\left(\frac{1}{6}, -\frac{\sqrt{35}}{6}\right)$

6. $P\left(\frac{\sqrt{7}}{4}, \frac{3}{4}\right)$

7. P is on the terminal side of $\theta = 45°$.

8. P is on the terminal side of $\theta = 120°$.

9. P is on the terminal side of $\theta = 240°$.

10. P is on the terminal side of $\theta = 330°$.

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13-6 Study Guide and Intervention *(continued)*

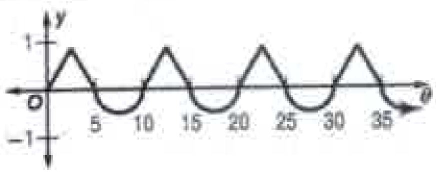
Circular Functions

Periodic Functions

A **periodic function** has y-values that repeat at regular intervals. One complete pattern is called a **cycle**, and the horizontal length of one cycle is called a **period**.

The sine and cosine functions are periodic; each has a period of 360° or 2π radians.

Example 1 **Determine the period of the function.**



The pattern of the function repeats every 10 units, so the period of the function is 10.

Example 2 **Find the exact value of each function.**

a. sin 855°

$\sin 855° = \sin(135° + 720°)$

$= \sin 135°$ or $\frac{\sqrt{2}}{2}$

b. $\cos\left(\frac{31\pi}{6}\right)$

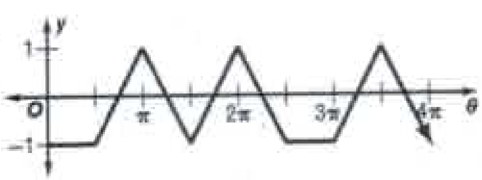
$\cos\left(\frac{31\pi}{6}\right) = \cos\left(\frac{7\pi}{6} + 4\pi\right)$

$= \cos\frac{7\pi}{6}$ or $-\frac{\sqrt{3}}{2}$

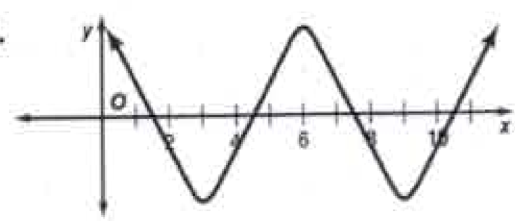
Exercises

Determine the period of each function.

1.



2.



Find the exact value of each function.

3. $\sin(-510°)$

4. $\sin 495°$

5. $\cos\left(-\frac{5\pi}{2}\right)$

6. $\sin\left(\frac{5\pi}{3}\right)$

7. $\cos\left(\frac{11\pi}{4}\right)$

8. $\sin\left(-\frac{3\pi}{4}\right)$

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13-7 Study Guide and Intervention

Graphing Trigonometric Functions

Sine, Cosine, and Tangent Functions Trigonometric functions can be graphed on the coordinate plane. Graphs of periodic functions have repeating patterns, or *cycles*; the horizontal length of each cycle is the *period*. The **amplitude** of the graph of a sine or cosine function equals half the difference between the maximum and minimum values of the function. Tangent is a trigonometric function that has asymptotes when graphed.

Sine, Cosine, and Tangent Functions	Parent Function	$y = \sin \theta$	$y = \cos \theta$	$y = \tan \theta$
	Domain	{all real numbers}	{all real numbers}	$\{\theta \mid \theta \neq 90 + 180n, n \text{ is an integer}\}$
	Range	$\{y \mid -1 \le y \le 1\}$	$\{y \mid -1 \le y \le 1\}$	{all real numbers}
	Amplitude	1	1	undefined
	Period	360°	360°	180°

Example **Find the amplitude and period of each function. Then graph the function.**

a. $y = 4 \cos \frac{\theta}{3}$

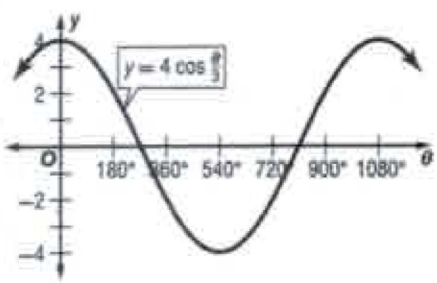
First, find the amplitude.

$|a| = |4|$, so the amplitude is 4.

Next find the period.

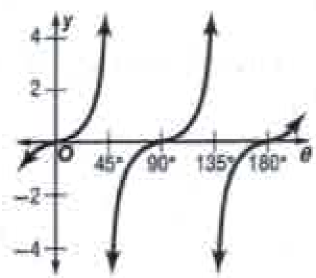
$\frac{360°}{\left|\frac{1}{3}\right|} = 1080°$

Use the amplitude and period to help graph the function.



b. $y = \frac{1}{2} \tan 2\theta$

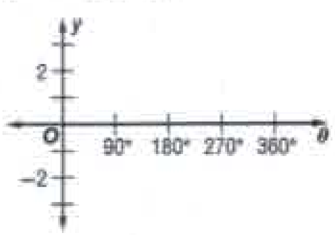
The amplitude is not defined, and the period is 90°.



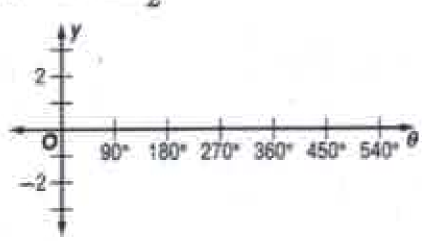
Exercises

Find the amplitude, if it exists, and period of each function. Then graph the function.

1. $y = -4 \sin \theta$



2. $y = 2 \tan \frac{\theta}{2}$



13-7 Study Guide and Intervention *(continued)*

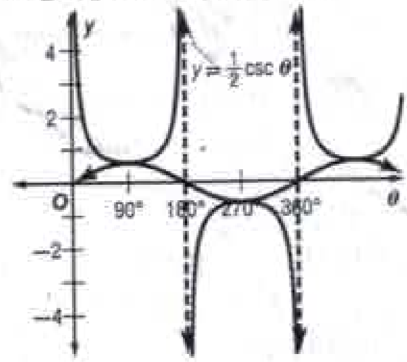
Graphing Trigonometric Functions

Graphs of Other Trigonometric Functions The graphs of the cosecant, secant, and cotangent functions are related to the graphs of the sine, cosine, and tangent functions.

	Parent Function	$y = \csc\theta$	$y = \sec\theta$	$y = \cot\theta$
Cosecant, Secant, and Cotangent Functions	Domain	$\{\theta \mid \theta \neq 180n,\ n$ is an integer$\}$	$\{\theta \mid \theta \neq 90 + 180n,\ n$ is an integer$\}$	$\{\theta \mid \theta \neq 180n,\ n$ is an integer$\}$
	Range	$\{y \mid -1 > y$ or $y > 1\}$	$\{y \mid -1 > y$ or $y > 1\}$	{all real numbers}
	Amplitude	undefined	undefined	undefined
	Period	360°	360°	180°

Example **Find the period of $y = \frac{1}{2}\csc\theta$. Then graph the function.**

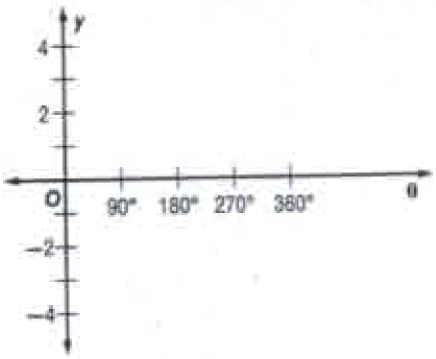
Since $\frac{1}{2}\csc\theta$ is a reciprocal of $\frac{1}{2}\sin\theta$, the graphs have the same period, 360°. The vertical asymptotes occur at the points where $\frac{1}{2}\sin\theta = 0$. So, the asymptotes are at $\theta = 0°$, $\theta = 180°$, and $\theta = 360°$. Sketch $y = \frac{1}{2}\sin\theta$ and use it to graph $y = \frac{1}{2}\csc\theta$.



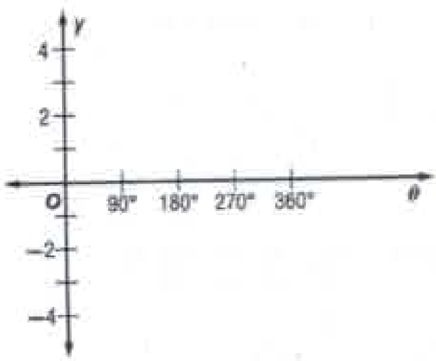
Exercises

Find the period of each function. Then graph the function.

1. $y = \cot 2\theta$



2. $y = \sec 3\theta$



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Study Guide and Intervention

Translations of Trigonometric Graphs

Horizontal Translations When a constant is subtracted from the angle measure in a trigonometric function, a **phase shift** of the graph results.

Phase Shift	The phase shift of the graphs of the functions $y = a \sin b(\theta - h)$, $y = a \cos b(\theta - h)$, and $y = a \tan b(\theta - h)$ is h, where $b > 0$. If $h > 0$, the shift is h units to the right. If $h < 0$, the shift is h units to the left.

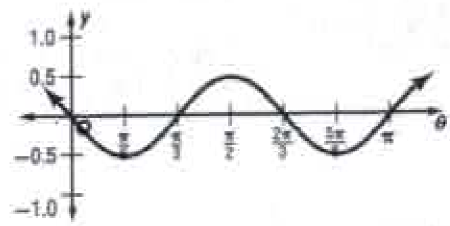
Example **State the amplitude, period, and phase shift for $y = \frac{1}{2} \cos 3\left(\theta - \frac{\pi}{2}\right)$. Then graph the function.**

Amplitude: $|a| = \left|\frac{1}{2}\right|$ or $\frac{1}{2}$

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|3|}$ or $\frac{2\pi}{3}$

Phase Shift: $h = \frac{\pi}{2}$

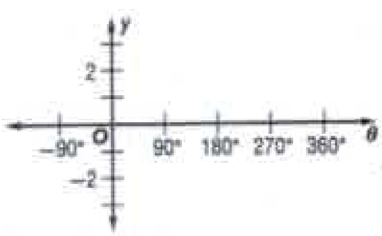
The phase shift is to the right since $\frac{\pi}{2} > 0$.



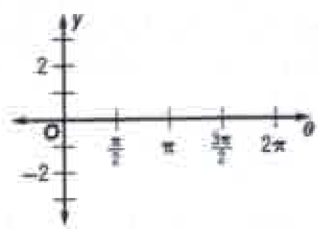
Exercises

State the amplitude, period, and phase shift for each function. Then graph the function.

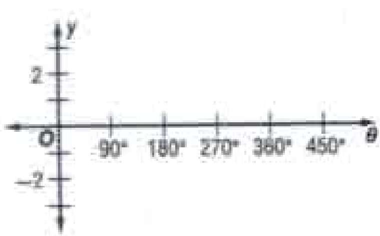
1. $y = 2 \sin (\theta + 60°)$



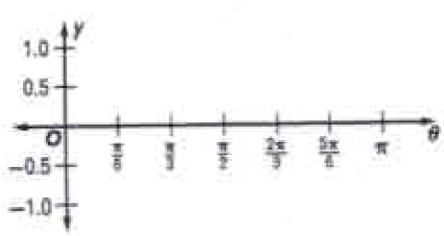
2. $y = \tan \left(\theta - \frac{\pi}{2}\right)$



3. $y = 3 \cos (\theta - 45°)$



4. $y = \frac{1}{2} \sin 3\left(\theta - \frac{\pi}{3}\right)$



13-8 Study Guide and Intervention *(continued)*

Translations of Trigonometric Graphs

Vertical Translations When a constant is added to a trigonometric function, the graph is shifted vertically.

Vertical Shift	The vertical shift of the graphs of the functions $y = a \sin b(\theta - h) + k$, $y = a \cos b(\theta - h) + k$, and $y = a \tan b(\theta - h) + k$ is k. If $k > 0$, the shift is k units up. If $k < 0$, the shift is k units down.

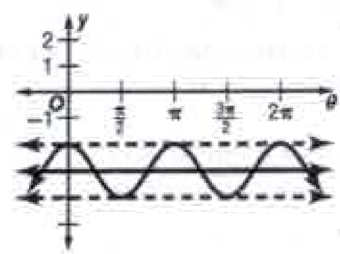
The **midline** of a vertical shift is $y = k$.

Graphing Trigonometric Functions	**Step 1** Determine the vertical shift, and graph the midline. **Step 2** Determine the amplitude, if it exists. Use dashed lines to indicate the maximum and minimum values of the function. **Step 3** Determine the period of the function and graph the appropriate function. **Step 4** Determine the phase shift and translate the graph accordingly.

Example **State the amplitude, period, vertical shift, and equation of the midline for $y = \cos 2\theta - 3$. Then graph the function.**

Amplitude: $|a| = |1|$ or 1

Period: $\frac{2\pi}{|b|} = \frac{2\pi}{|2|}$ or π



Vertical Shift: $k = -3$, so the vertical shift is 3 units down.

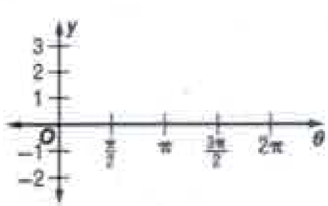
The equation of the midline is $y = -3$.

Since the amplitude of the function is 1, draw dashed lines parallel to the midline that are 1 unit above and below the midline. Then draw the cosine curve, adjusted to have a period of π.

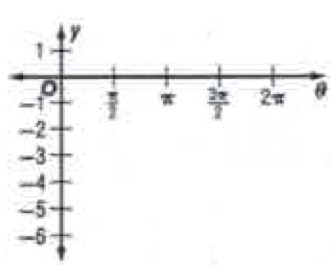
Exercises

State the amplitude, period, vertical shift, and equation of the midline for each function. Then graph the function.

1. $y = \frac{1}{2} \cos \theta + 2$



2. $y = 3 \sin \theta - 2$



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13-9 Study Guide and Intervention

Inverse Trigonometric Functions

Inverse Trigonometric Functions If you know the value of a trigonometric function for an angle, you can use the *inverse* to find the angle. If you restrict the function's domain, then the inverse is a function. The values in this restricted domain are called **principal values.**

Principal Values of Sine, Cosine, and Tangent	$y = \text{Sin } x$ if and only if $y = \sin x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. $y = \text{Cos } x$ if and only if $y = \cos x$ and $0 \le x \le \pi$. $y = \text{Tan } x$ if and only if $y = \tan x$ and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
Inverse Sine, Cosine, and Tangent	Given $y = \text{Sin } x$, the inverse sine function is defined by $y = \text{Sin}^{-1} x$ or $y = \text{Arcsin } x$. Given $y = \text{Cos } x$, the inverse cosine function is defined by $y = \text{Cos}^{-1} x$ or $y = \text{Arccos } x$. Given $y = \text{Tan } x$, the inverse tangent function is given by $y = \text{Tan}^{-1} x$ or $y = \text{Arctan } x$.

Example 1 **Find the value of $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right)$. Write angle measures in degrees and radians.**

Find the angle θ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ that has a sine value of $\frac{\sqrt{3}}{2}$.

Using a unit circle, the point on the circle that has y-coordinate of $\frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$ or 60°.

So, $\text{Sin}^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or 60°.

Example 2 **Find $\tan\left(\text{Sin}^{-1} \frac{1}{2}\right)$. Round to the nearest hundredth.**

Let $\theta = \text{Sin}^{-1} \frac{1}{2}$. Then $\text{Sin } \theta = \frac{1}{2}$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. The value $\theta = \frac{\pi}{6}$ satisfies both conditions. $\tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$ so $\tan\left(\text{Sin}^{-1} \frac{1}{2}\right) = \frac{\sqrt{3}}{3}$.

Exercises

Find each value. Write angle measures in degrees and radians.

1. $\text{Cos}^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

3. $\text{Arccos}\left(-\frac{1}{2}\right)$

4. $\text{Arctan } \sqrt{3}$

5. $\text{Arccos}\left(-\frac{\sqrt{2}}{2}\right)$

6. $\text{Tan}^{-1}(-1)$

Find each value. Round to the nearest hundredth if necessary.

7. $\cos\left[\text{Sin}^{-1}\left(-\frac{\sqrt{2}}{2}\right)\right]$

8. $\tan\left[\text{Arcsin}\left(-\frac{5}{7}\right)\right]$

9. $\sin\left(\text{Tan}^{-1} \frac{5}{12}\right)$

10. $\text{Cos}\,[\text{Arcsin}\,(-0.7)]$

11. $\cos\,(\text{Arctan } 5)$

12. $\sin\,(\text{Cos}^{-1}\, 0.3)$

13-2 Study Guide and Intervention *(continued)*

Inverse Trigonometric Functions

Solve Equations by Using Inverses You can rewrite trigonometric equations to solve for the measure of an angle.

Example **Solve the equation Sin $\theta = -0.25$. Round to the nearest tenth if necessary.**

The sine of angle θ is -0.25. This can be written as Arcsin$(-0.25) = \theta$.

Use a calculator to solve.

KEYSTROKES: [2nd] [SIN⁻¹] [(−)] .25 [ENTER] −14.47751219

So, $\theta \approx -14.5°$

Exercises

Solve each equation. Round to the nearest tenth if necessary.

1. Sin $\theta = 0.8$

2. Tan $\theta = 4.5$

3. Cos $\theta = 0.5$

4. Cos $\theta = -0.95$

5. Sin $\theta = -0.1$

6. Tan $\theta = -1$

7. Cos $\theta = 0.52$

8. Cos $\theta = -0.2$

9. Sin $\theta = 0.35$

10. Tan $\theta = 8$