Geometry, Periods 4 & 8 – Word for March 16-27 – Dr. Sargent

I have designed this two weeks of work primarily for those who have access to ALEKS. If you are doing this with pencil and paper, some of the material will be a bit more difficult for you to accomplish. Occasionally, I have substituted a similar but different assignment for those doing this with pencil and paper. Please note that all ALEKS assignments in this unit expire at the end of Friday, March 27, so you will need to be doing these before then.

Days 1 & 2 – Finishing Unit 8

You will recall that we were reviewing for a test on Unit 8. The test is online on ALEKS. I am going to enable the test, with a maximum of three times taking it. You may use your first time taking it as a practice, then go back and review what you have done. Then you may take it for real a maximum of two additional times. I will record your highest grade. You will have 90 minutes to complete each test once you have begun. Do not leave the test and go back or it will assume you were finished. The test will be open and available through Sunday night, March 22. If you need an extension, please ask, stating why.

If you are receiving the work in written form, I am enclosing two copies of the test. The first, #5, will contain answers so you can review the material. The second test, #1, will be the actual test you will need to take. You should then submit the test back to me, and I will grade it. If you wish to take it again, just let me know and I will provide a new version of the test. I am counting on you to be honest in taking this test – that is, that you will do the work yourself without using internet helps.

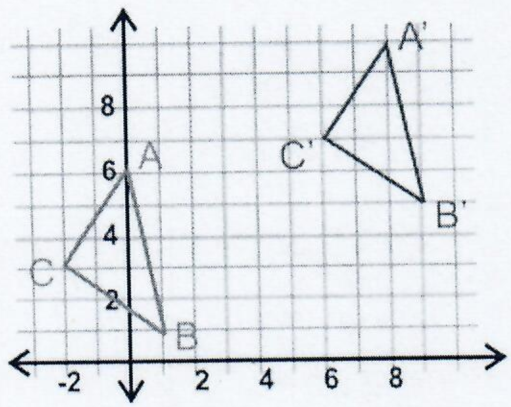
Day 3-5 – Transformations: Translations and Reflections

I have provided brief explanations of the new concepts being introduced beginning Day 3 with the unit on transformations. If you need additional explanations, I recommend the ALEKS examples and illustrations, or Khan Academy. The first video in the Khan Academy series is here: https://www.khanacademy.org/math/geometry/hs-geo-transformations . From there, you can begin accessing additional videos on the various types of transformations.

This is going to be difficult to explain without being able to demonstrate, but I am going to try. A tranformation involves moving an image or transforming an image to a new location. We call the first image, or the original image, the pre-image, and the resulting transformation, the new figure, is called the image. A transformation can change the size, position, or even the shape of an object.

The first type of transformation is simply a **translation**. In a translation, we simply move and image right or left, up or down, on the coordinate plane. If the original pre-image is A, then the resulting new image would be called A' (read as "A Prime"). If there is an additional image of A', it would be read as A" (read as "A double prime").

Let's look at a translation:



In this translation, the original preimage (ΔABC) has been translated eight units to the right and 4 units up, resulting in the translated image, ΔA'B'C'. Every vertex has been moved the same amount, right 8 and up 4. You could write this translation like this: (x,y) →(x+8, y+4).

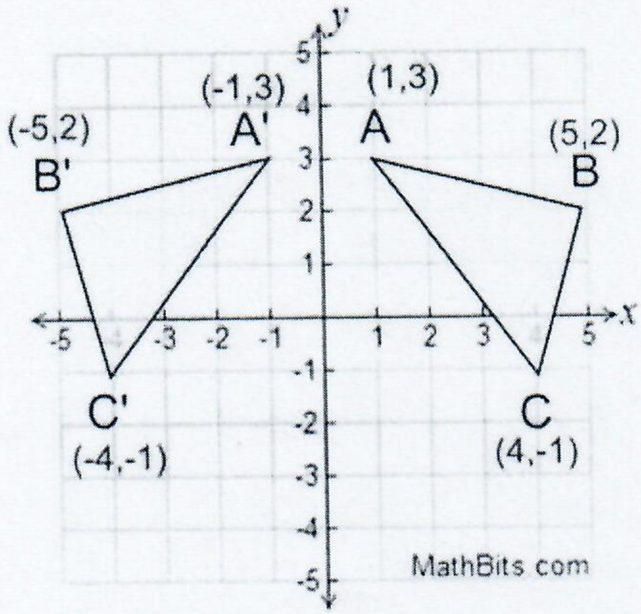
A translation, then, simply slides a figure horizontally (to the right or left) and/or vertically (up or down). In symbolic form, we would write it like this: (x,y) → (x + h, y + k) where h represents the horizontal shift and k represents the vertical shift. In the example above, h = 8 and k = 4. Every point on the preimage would be moved horizontally h units and vertically k units to form the new image.

I hope this helps explain. I have given you work on ALEKS titled "Homework 1 – Translations." Please complete this as soon as you can. I have left it open through March 27, but it really should be done earlier than this.

If you are doing written work, it will be difficult to ask for and receive feedback (though I am available each day through live grades or telephone), I have included the answers to the odd-numbered questions for the version of this work that I have printed out. However, please make every effort to do the problems yourself and turn to the answers only to check your understanding. You will not need to return this work to me.

A **reflection** is just a bit more complicated type of transformation than a translation. In a reflection, an object flips over a line called the line of reflection. Each point of the preimage is the same distance from the line of reflection as that reflected point in the image. Possible lines of reflection might be the x- or y-axis; a vertical or horizontal line (x = * or y = *), or a diagonal line (x = y or x = -y). We call this a reflection because it is a lot like looking at things reflected in a mirror.

Let's look at a reflection:



In this reflection, ΔABC is reflected to create image ΔA'B'C'. Note that the image is flipped as part of the reflection. Each point is the same distance from the line of reflection, the y-axis. Thus, A and A' are both 1 unit from y, B and B' are both 5 units from y, and C and C' are both 4 units from y. Note that when an image is reflected across the y-axis, the coordinates of y do not change, but the coordinates of x become the opposite. (If you reflect across the x-axis, the coordinates of x will not change, and the coordinates of y will become the opposite).

If you reflect a preimage across the diagonal line y = x, then the coordinates of the image become (y,x). If you reflect over the diagonal y = -x, then the resultant reflection's coordinates become (-y, -x). Note that these are not the only possible lines of reflection, but are some of the more common.

Please note that it is possible for a point or figure to be both translated and reflected.

I have provided a worksheet on ALEKS entitled, "Homework 2 – Reflections" which you should now do to gain some practice. This is also included in the written work for those doing this via pencil and paper. You will not need to return this work.

I have prepared a short quiz, "Quiz 1," on ALEKS on these two topics: translations and reflections, which you should now do. If you are doing the work with pencil and paper, please return this to me when you are finished. If you are doing this on ALEKS, you will not be able to leave the quiz once you begin or ALEKS will assume you are done and submit it for grading.

Day 6-7 – Rotations and Symmetry

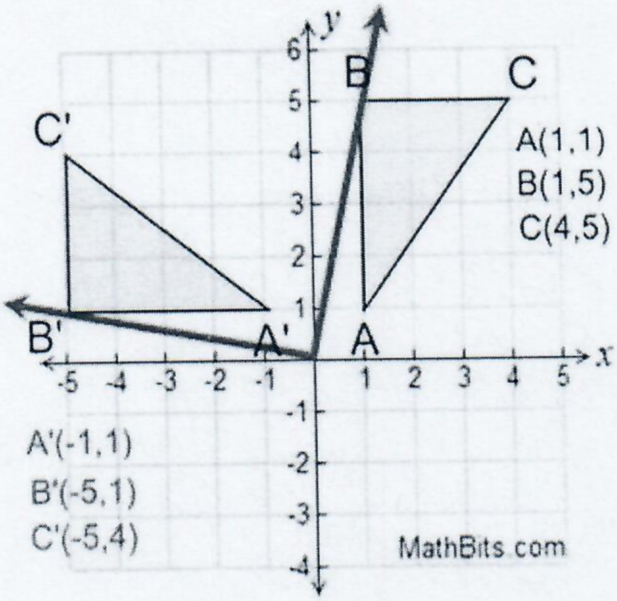
A third kind of transformation is called a **Rotation**. A rotation is simply a turn around a fixed point. A figure rotates at a specific angle and direction. If a figure rotates around the origin point (0,0), here are the rules for determining the new coordinates of the image formed by rotation:

If the figure rotates counterclockwise 90º, then (x,y) → (-y, x)

If the figure rotates counterclockwise 180º, then (x,y) → (-x,-y)

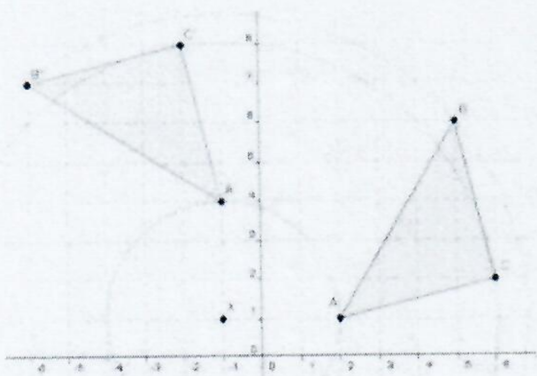
If the figure rotates counterclockwise 270º, then (x,y) → (y, -x)

Let's take a look at a rotation:



In this rotation, the figure has rotated 90⁰ counterclockwise around the point of origin. Not that point C (4,5) has become point C' (-5,4). The figure has not changed size or shape, it has simply rotated around a fixed point. It is possible for a figure to be rotated and translated, or even rotated and reflected.

Let's look at another rotation, below



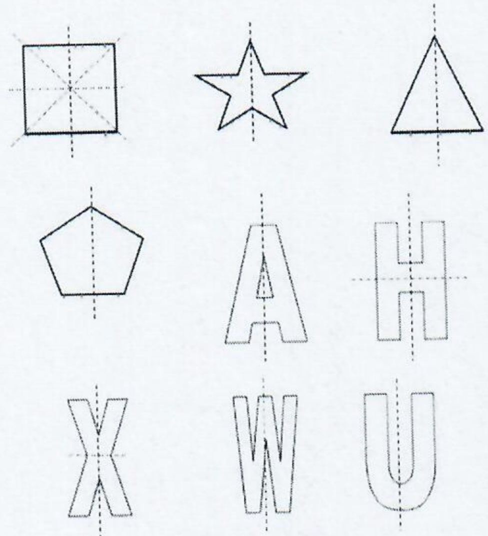
Notice than in this second rotation, which is also 90⁰ counterclockwise, the point of rotation is not the origin. Therefore, we cannot make the assumption that (x,y) → (y, -x). In fact, A was (2,1) and point A' is 4, -1). The neat rule mentioned earlier only applies when the point of rotation is the origin.

Note that while I have spoken of counterclockwise rotation, the direction can also be clockwise. Also notice that 90⁰ clockwise is the same as 270° counterclockwise, and vice versa, and that 180⁰ is the same in either direction.

I have provided you with "Homework 3 – Rotations" in ALEKS (or in written form). You will not need to submit this work to me.

Now let's look at the idea of **symmetry**, shall we? We say that figures have symmetry in three different ways:

1) Line Symmetry



Figures have line symmetry if the figure can be mapped onto itself by a reflection across a line. Think of a "mirror image." For example, an isosceles triangle can be mapped onto itself across a line by drawing a line through the vertex angle perpendicular to the base. Here to the left are some examples of line symmetry. The line that cuts through the figure and divides it into a mirror image is called the "line of symmetry."

2) Point Symmetry

Figures have point symmetry if the figure can be mapped onto itself by rotating it 180° around a center point. In other words, the figure looks the same upside down. The letters 'N' and 'S' are examples of this, as is an hour glass.

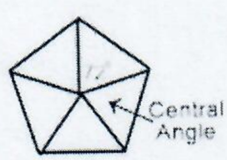
3) Rotational Symmetry.

A figure has rotational symmetry if the figure can be mapped onto itself by rotating the figure less than 360°. A pinwheel or a regular hexagram would be good examples of this.

Some figures will have more than one type of symmetry. For example, the capital letter H has two possible lines of symmetry, but also has point symmetry and rotational symmetry.

Remembering that a circle has 360° can help us figure out the angle of rotation necessary for a figure to rotate so that it is mapped upon itself. For example, visualize a regular pentagon. Each of the points lies 1/5 of the way around a circle drawn from the pentagon's center with a diameter just touching each vertex. Thus, each vertex would need to be rotated 72° to map onto the next vertex (360/5 = 72). Here is an illustration of this:

$$\frac{360}{5} = 72^{\circ}$$



I have created an assignment for you on ALEKS to practice this – "Homework 4 – Symmetry." You will not need to submit this if doing it with pencil and paper.

Day 8 – Congruence Transformations

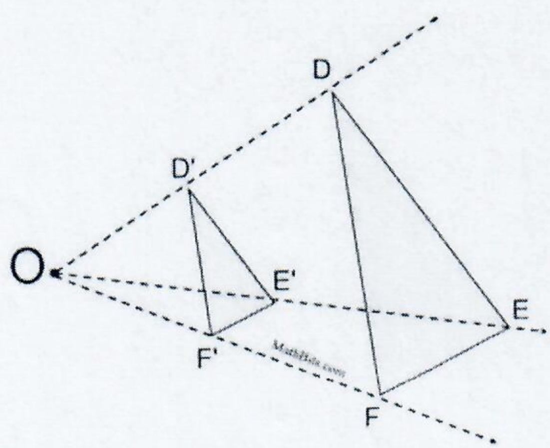
I have prepared Homework 5 – Congruence Transformations on ALEKS. This is homework that requires you to put together what you have learned about transformations up to this point. This should be completed and submitted to me for grading. (If you do it on ALEKS, I will see it, of course. Those doing it in writing will need to return it).

Day 9-10 – Dilations

A dilation is the enlargement or reduction of a figure in size, while retaining the shape. This should be familiar to us, for it is the same principle as similar figures. The scale factor indicates how much the figure will enlarge or reduce. If k is the scale factor, then:

When $k > 1$, the dilation is an enlargement; and when $k < 1$, the dilation is a reduction. If $k = 1$, then the images are congruent.

We almost always determine a dilation based on the origin as the center. This means, we figure that if A is x units from the origin (the center) with a scale factor of 1/2, then A' will be ½ x units from the origin along the same line from the origin. Example: OD' = ½ OD



For day 9 I have prepared "Homework 6 – Dilations" on ALEKS so that you might gain practice working with dilations. For those of you doing this on pencil and paper, I have provided an alternative assignment, the sheet called "Dilations."

For day 10, I recommend going back over Days 3-9 in preparation for a Test on this material when we return after Spring Break. (Or, if we are still out, when we send you the next assignments).

ALEKS TOPICS – Please note that your regular assignment of 12 topics per week continues. The week of Spring Break, March 30-Apr 5, is a bonus week of topics.