

Geometry, Period 5 – Work for April 13-24 – Dr Sargent

Week of April 13-17 – Angle Measures in Polygons

This week we begin a unit on Polygons. The word “poly” means “many,” so a polygon is simply a many-sided geometric figure. I am attaching a worksheet on Polygons. The first sentence should be completed to read: A polygon is a closed figure formed by three or more line segments called sides.

As you move down the page, you will see shapes beginning with a 3-sided figure and growing incrementally by one additional side as you move down the page. Remember that the angle measures in a triangle sum to 180° , so if you add the number of triangles in a figure and multiply this by 180 you will obtain the total of the angle measures in a polygon. Hence, a triangle has 3 sides, contains only one triangle, and the sum of the angles is 180° . The next figure, a quadrilateral, has 4 sides, can be divided into two triangles, and the total of the angle measures is $2 * 180 = 360^\circ$. And continuing down the page, you can continue to fill in the blanks. Note that we name polygons up to about 10 sides as listed. After ten, we simply begin to use numbers, like an 11-gon for an 11-sided polygon, or 20-gon for a polygon with 20 sides.

If you complete the chart, you will notice a pattern. Each figure can be divided into a number of triangles equal to the number of sides minus 2. So, a hexagon can be divided into $6-2$ or 4 triangles. Thus, we can derive a formula for the sum (S) of the angles in any polygon: $S = (n - 2)180$, where n is the number of sides in the polygon. So, working on the worksheet, problem #1, find the sum of the measures of the interior angles in a 15-gon, we can write the formula as follows: $S = (15 - 2)180 = (13)180 = 2340^\circ$. See if you can now complete problems 2-4.

On the back of the worksheet, a regular polygon is: “A polygon in which all side measures are congruent, therefore all angle measures are congruent.” To find any one angle measure, then, we simply divide S by n. In problem 5, for example, we are asked to find the angle measure of each angle in a regular pentagon (By the way, you should memorize the names of polygons that have up to 10 sides). We use the formula $\frac{S}{n}$ which in this case is: $\frac{(5-2)180}{5} = \frac{540}{5} = 108^\circ$. See if you can do problem #6.

When we begin talking about the exterior angles of a polygon, remember that an exterior angle is formed by extending one side to form an angle outside the triangle. Notice also that this angle will be supplementary to the interior angle (since they form a linear pair). Thus, in the triangle, the exterior angle to $79^\circ = 180-79 = 101^\circ$. The angle exterior to 40° is $140^\circ (180-40)$, and the angle exterior to 61° is 119° . Add these together and you will discover the total of all the exterior angles is 360° . Now do the same for the quadrilateral, the pentagon, and the hexagon. What did you discover?

Yes, you are correct (hopefully you got it, right?), that all the exterior angles add up to 360° regardless of how many sides the polygon has. Thus, we can postulate that: "The sum of the exterior angles of any polygon is 360° ."

Now, having discovered this general rule, we can say that the measure of each exterior angle of a regular polygon is $360/n$. So, for problem number 7, the measure of each exterior angle of a hexagon is $360/6$ or 60° . See if you can complete the rest of the problems on this page.

Did you figure out how to do problems 9 and 10? Just substitute x in place of the number of sides. Thus, in problem 9, we are given that each exterior angle in a regular polygon measures 12° , so the formula would look like this: $\frac{360}{x} = 12$; thus $360 = 12x$; and $x = 30$. There are, therefore, 30 sides in the regular polygon whose exterior angles each measure 12° .

I am attaching a worksheet giving you more practice problems. The formulae across the top are:

Sum of interior angle measures: $S = (n - 2)180$

Sum of the exterior angle measures: 360° .

Interior angle measures of a regular polygon: $\frac{S}{n}$

Exterior angle measures of a regular polygon: $\frac{360}{n}$

Number of sides of a regular polygon: $360/\text{exterior angle measurement}$

Now, see if you can do the problems on this worksheet. I am providing answers to selected problems at the end of these instructions so you can check your work, but please try and do the problems yourself before looking at these answers. You do NOT need to return this work to me – this is for your practice.

Then, see if you can complete Unit 7 Homework 1, also attached. I have included some selected answers at the end of this sheet, so you can check yourself, but once again, please try it yourself before checking answers. When you have finished the homework, you can return it to me either by taking a picture and attaching to a message; by scanning and attaching to a message; or by placing it in the box at the school between 9-11am any school day. If you return the complete paper copy, please make sure both your name and mine is on the work you return.

Week of April 20-24 – Parallelograms

I have attached a worksheet on the topic of Parallelograms. Please get that in front of you at this time. A parallelogram is "a quadrilateral in which both pairs of opposite sides are parallel." A parallelogram has five properties, which you should memorize:

- Opposite sides are parallel
- Opposite side are congruent
- Opposite angles are congruent
- Consecutive angles are supplementary
- Diagonals bisect each other

Once you have these properties clearly in mind, you can find missing measures in parallelograms with relative ease. Take problem 1, for example, if $BC = 8$, then $AD = 8$ because opposite sides are congruent. Similarly, if $AB = 15$, then $CD = 15$. If $m\angle D = 68^\circ$, then $m\angle A = 112^\circ$, because consecutive angles are supplementary ($180 - 68 = 112$). The same is true for $m\angle C$. Then the $m\angle B = 68^\circ$ because opposite angles are congruent. So you get the idea. See if you can begin to complete the rest of this page. Remember that since opposite sides are parallel, you will have opposite interior angles congruent when you begin working with the diagonals in problem #4.

The next page begins substituting algebraic expressions for numbers, but the same properties of parallelograms still apply. For example, $LM = PN$ in problem 7, because they are opposite sides and opposite sides are congruent. Thus, you can set up the problem like this: $9x - 25 = 5x + 7$ and then solve for x . I have included the answers for selected problems at the end of these instructions, so you can begin to check yourself for understanding. You do NOT need to return this work to me.

Proving Parallelograms on the Coordinate Plane

The next worksheet in this packet involves learning how to prove a quadrilateral is a parallelogram when you are given its coordinates on the coordinate plane. Essentially, there are three ways to prove a figure is a parallelogram:

Method 1: Prove that both pairs of sides are congruent. To do this, you use the distance formula. Basically, in the example given, if $\overline{AB} \cong \overline{CD}$ AND $\overline{DA} \cong \overline{BC}$, then the figure is a parallelogram. So you must use the distance formula on each side to see if the opposite sides are the same length. Remember that the distance formula is: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Method 2: Prove that both pairs of opposite sides are parallel. To do this, you would use the slope formula. Basically, if $\overline{AB} \parallel \overline{CD}$, AND $\overline{AD} \parallel \overline{BC}$, then both pair of opposite sides are parallel, and you have a parallelogram. Remember that the slope formula is: $m = \frac{y_2 - y_1}{x_2 - x_1}$. You will need to apply this formula to all four sides to see if opposite sides have the same slope.

Method 3: Prove that one pair of opposite sides is both congruent and parallel. Basically, if $\overline{AB} \cong \overline{CD}$ AND $\overline{AB} \parallel \overline{CD}$, then one pair of sides is both congruent and parallel, so you have a parallelogram. To prove this, you will need to apply the distance and slope formulae to the same pair of opposite sides.

In problems 1 & 2, then, you would apply the distance formula to all 4 sides to see if both sets of opposite sides are congruent. In problems 3 & 4, you would apply the slope formula to all four sides to see if the opposite sides have the same slope and are, thus, parallel. In problems 5 & 6 you will apply both the distance and slope formulae to two sides that are opposite (AB & CD, for example) and if this set of sides is both congruent and parallel, then you have a parallelogram. See if you can determine which figures are parallelograms. You will NOT need to return this work to me.

When you have a grasp of this, please complete Unit 7 Homework 2, which you can then return to me for grading.

Ongoing ALEKS assignment – Please remember that your weekly assignment of 12 topics on ALEKS continues.

Answers to Selected Questions

Interior and Exterior Angle Measures

Problem 1: $S = (5 - 2)180 = 540^\circ$

Problem 3: $S = (8 - 2)180 = 1080; \frac{1080}{8} = 135^\circ$

Problem 7: $(n - 2)180 = 1620; n - 2 = \frac{1620}{180} = 22; n = 22 + 2 = 24 \text{ sides}$

Problem 9: $\frac{360}{20} = 18^\circ$

Problem 11: If the interior angles measure 108° , then the measure of the exterior angle is 72° .

You can then set it up this way: $\frac{360}{n} = 72; 360 = 72n; n = 5 \text{ sides}$

Problem 13: $148 + 140 + 136 + x + 142 + 150 + 90 = 900; x + 806 = 900; x = 94^\circ$

Problem 15: $8x + 7 + 5x + 18 + 10x + 13 + 14x - 11 = 360; 37x + 27 = 360; 37x = 333; x = 9; m\angle B = 8(9) + 7 = 79^\circ$

Unit 7 Homework 1

Problem 1: 1080°

Problem 4: 157.5°

Problem 7: 16

Problem 10: $(n - 2)180 = 5400$; $n - 2 = 30$; $n = 32$

Problem 13: $S = (8 - 2)180 = 1080$; $x + 129 + 132 + 136 + 124 + 141 + 158 + 116 = 1080$; $x + 936 = 1080$;
 $x = 144^\circ$

Problem 16: $S = (6 - 2)180 = 720$; $21x + 321 = 720$; $21x = 399$; $x = 19$; $m\angle V = 5(19) + 8 = 103^\circ$

Unit 7 Homework 2

Problem 1: $MN = 31$; $KN = 45$; $m\angle K = 61^\circ$; $m\angle L = 119^\circ$; $m\angle M = 61^\circ$

Problem 4: $12x - 22 = 7x - 2$; $5x = 20$; $x = 4$; $KL = 7(4) - 2 = 26$

Problem 7: $10x - 27 = 2x + 29$; $8x = 56$; $x = 7$; $m\angle Y = 10(7) - 27 = 43^\circ$; $m\angle V = 180 - 43 = 137^\circ$

Problem 10:

$QR = \sqrt{(-10 - 1)^2 + (-2 + 1)^2} = \sqrt{121 + 1} = \sqrt{122}$; $ST = \sqrt{(1 + 11)^2 + (-7 + 8)^2} = \sqrt{144 + 1} = \sqrt{145}$. Therefore, since this pair of opposite sides is not congruent, then QRST cannot be a parallelogram, so we do not have to check further.

Name:

Date:

Topic:

Class:

Main Ideas/Questions


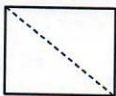
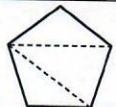
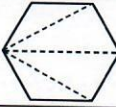
Notes/Examples

POLYGON

A **polygon** is a _____ figure formed by three or more _____, called _____.

Sum of the
INTERIOR ANGLE
Measures

The sum of the measures of the interior angles in any polygon can be determined by the number of triangles that can be drawn within the polygon. Complete the table below and look for a pattern to find the sum of the degrees in any polygon.

Polygon	Picture	# of Sides	# of Triangles	Sum of Interior \angle 's
Triangle				
Quadrilateral				
Pentagon				
Hexagon				
Heptagon	X			
Octagon	X			
Nonagon	X			
Decagon	X			

INTERIOR
Angle Sum
FORMULA

If n represents the number of sides of a polygon, then the sum of the interior angle, S , can be found using the formula:

Find the sum of the measures of the interior angles in each polygon.

1. 15-gon

2. 21-gon

3. 48-gon

4. 36-gon

REGULAR POLYGON

The measure of a single interior angle in a regular polygon can be found by dividing the sum of the interior angle measures, S , by the number of sides, n .

Find the measure of each interior angle in the following polygons.

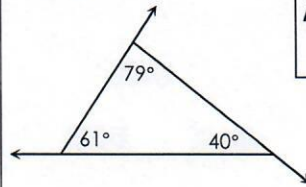
5. regular pentagon

6. regular 18-gon

Sum of the EXTERIOR ANGLE Measures

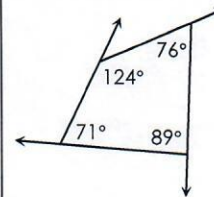
Exterior angles are supplementary to their adjacent interior angle. Find the measure of each exterior angle on the polygons below, then give the sum of all exterior angle measures.

Triangle:



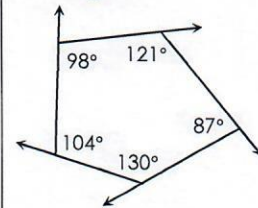
Sum of Exterior Angles Measures:

Quadrilateral:



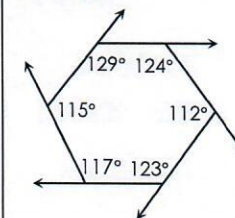
Sum of Exterior Angles Measures:

Pentagon:



Sum of Exterior Angles Measures:

Hexagon:



Sum of Exterior Angles Measures:

What can you conclude about the sum of the exterior angles measures of a polygon?

MORE EXAMPLES

7. What is the measure of each exterior angle of a regular hexagon?

8. What is the measure of each exterior angle of a regular 24-gon?

9. If the exterior angle of a regular polygon measures 12° , how many sides does the polygon have?

10. If the exterior angle of a regular polygon measures 40° , how many sides does the polygon have?

more practice with

INTERIOR & EXTERIOR ANGLE MEASURES

INTERIOR & EXTERIOR OF ANY POLYGON REFERENCE:

Sum of the INTERIOR Angle Measures:	Sum of the EXTERIOR Angle Measure:
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INTERIOR & EXTERIOR ANGLES OF REGULAR POLYGONS REFERENCE:

Interior Angle Measure of a Regular Polygon:	Exterior Angle Measure of a Regular Polygon:	The Number of Sides of a Regular Polygon:
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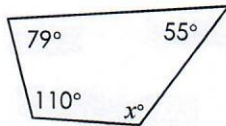
PRACTICE QUESTIONS

1. What is the sum of the measures of the interior angles of a pentagon?	2. What is the sum of the measures of the interior angles of a 27-gon?
3. What is the measure of each interior angle of a regular octagon?	4. What is the measure of each interior angle of a regular 20-gon?
5. Five angles of a hexagon measure 119° , 129° , 104° , 139° , and 95° . What is the measure of the sixth angle?	
6. The sum of the interior angles of a polygon is 1620° . How many sides does the polygon have?	
7. The sum of the interior angles of a polygon is 3960° . How many sides does the polygon have?	
8. What is the sum of the measures of the exterior angles of a nonagon?	9. What is the measure of each exterior angle of a regular 20-gon?

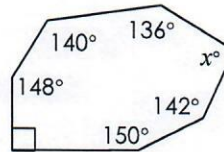
10. If the exterior angle of a regular polygon measures 9° , how many sides does the polygon have?

11. If the interior angle of a regular polygon measures 108° , how many sides does the polygon have?

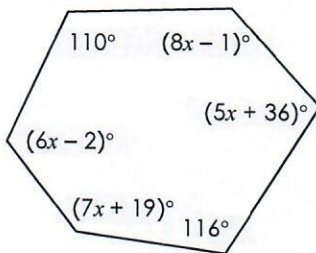
12. Find the value of x .



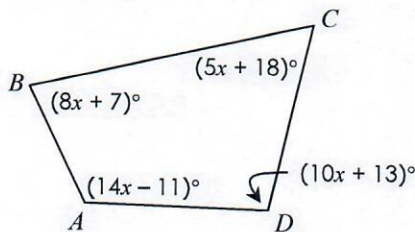
13. Find the value of x .



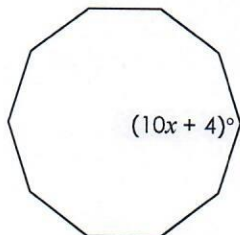
14. Solve for x .



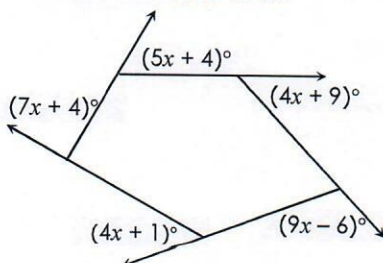
15. Find $m\angle B$.



16. If the figure below is a regular polygon, find the value of x .



17. Find the value of x .



Name: _____

Unit 7: Polygons & Quadrilaterals



Date: _____ Per: _____

Homework 1: Angles of Polygons

**** This is a 2-page document! ****

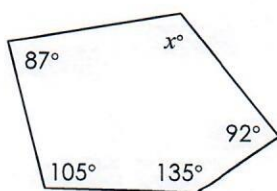
1. What is the sum of the measures of the interior angles of an octagon? _____
2. What is the sum of the measures of the interior angles of a 25-gon? _____
3. What is the measure of each interior angle of a regular hexagon? _____
4. What is the measure of each interior angle of a regular 16-gon? _____
5. What is the sum of the measures of the exterior angles of a decagon? _____
6. What is the measure of each exterior angle of a regular 30-gon? _____
7. An exterior angle of a regular polygon measures 22.5° . How many sides does it have? _____
8. An interior angle of a regular polygon measures 170° . How many sides does it have? _____

9. If the sum of the measures of the interior angles of a polygon is 1980° , how many sides does the polygon have?

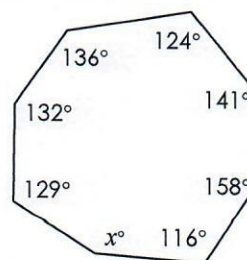
10. If the sum of the measures of the interior angles of a polygon is 5400° , how many sides does the polygon have?

11. The measure of the seven angles in a nonagon measure 138° , 154° , 145° , 132° , 128° , 147° , and 130° . If the two remaining angles are equal in measure, what is the measure of each angle?

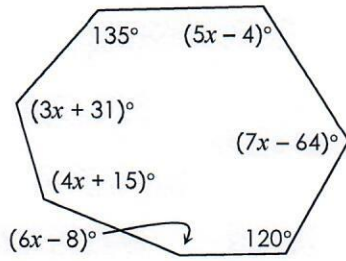
12. Find the value of x .



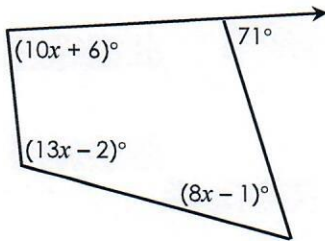
13. Find the value of x .



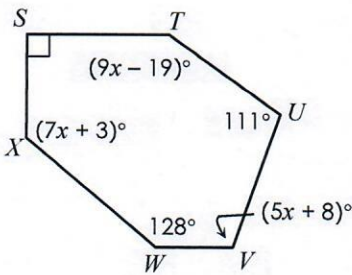
14. Find the value of x .



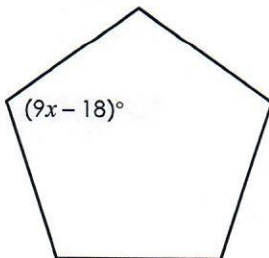
15. Find the value of x .



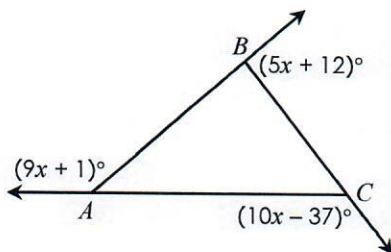
16. Find $m\angle V$.



17. If the figure below is a regular polygon, find the value of x .



18. Find $m\angle BCA$.



Name: _____

Date: _____

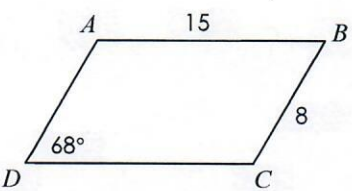
Topic: _____

Class: _____

Main Ideas/Questions	Notes/Examples
<p><i>Properties of</i> PARALLELOGRAMS</p>	Definition of a Parallelogram:
	Other important properties of parallelograms:
	① _____
	② _____
③ _____	
④ _____	

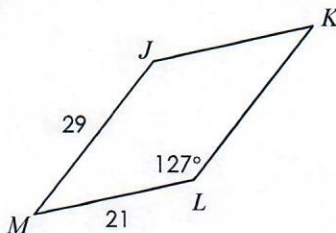
Directions: Each quadrilateral below is a parallelogram. Find the missing measures.

1.



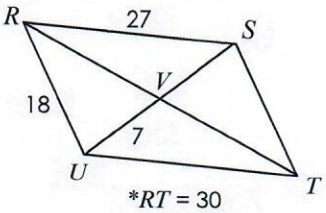
$AD = \underline{\hspace{2cm}}$
 $DC = \underline{\hspace{2cm}}$
 $m\angle A = \underline{\hspace{2cm}}$
 $m\angle B = \underline{\hspace{2cm}}$
 $m\angle C = \underline{\hspace{2cm}}$

2.



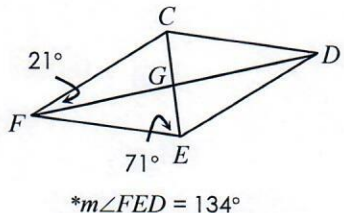
$JK = \underline{\hspace{2cm}}$
 $KL = \underline{\hspace{2cm}}$
 $m\angle J = \underline{\hspace{2cm}}$
 $m\angle K = \underline{\hspace{2cm}}$
 $m\angle M = \underline{\hspace{2cm}}$

3.



$UT = \underline{\hspace{2cm}}$
 $ST = \underline{\hspace{2cm}}$
 $VS = \underline{\hspace{2cm}}$
 $VT = \underline{\hspace{2cm}}$

4.



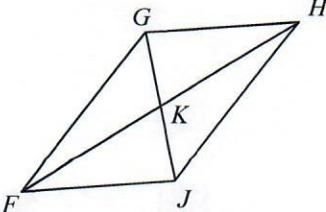
$m\angle DEC = \underline{\hspace{2cm}}$
 $m\angle CDE = \underline{\hspace{2cm}}$
 $m\angle ECD = \underline{\hspace{2cm}}$
 $m\angle DFE = \underline{\hspace{2cm}}$

5. Given $XY = 15$, $WX = 22$, $ZX = 32$, $WT = 10$, $m\angle WZY = 62^\circ$, $m\angle WXT = 27^\circ$, and $m\angle ZWT = 77^\circ$.



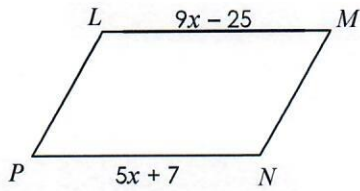
$ZW = \underline{\hspace{2cm}}$ $m\angle TZY = \underline{\hspace{2cm}}$
 $ZY = \underline{\hspace{2cm}}$ $m\angle XYZ = \underline{\hspace{2cm}}$
 $TX = \underline{\hspace{2cm}}$ $m\angle XWT = \underline{\hspace{2cm}}$
 $WY = \underline{\hspace{2cm}}$ $m\angle XYT = \underline{\hspace{2cm}}$

6. Given $m\angle GHF = 34^\circ$, $m\angle HJF = 124^\circ$, and $m\angle FKJ = 79^\circ$.

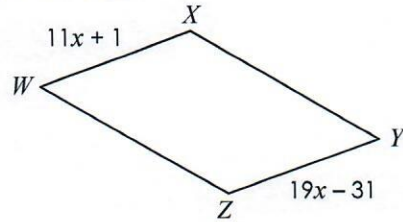


$m\angle GFJ = \underline{\hspace{2cm}}$ $m\angle JGH = \underline{\hspace{2cm}}$
 $m\angle FGH = \underline{\hspace{2cm}}$ $m\angle FGJ = \underline{\hspace{2cm}}$
 $m\angle HFJ = \underline{\hspace{2cm}}$ $m\angle FHJ = \underline{\hspace{2cm}}$
 $m\angle HKJ = \underline{\hspace{2cm}}$ $m\angle GJF = \underline{\hspace{2cm}}$

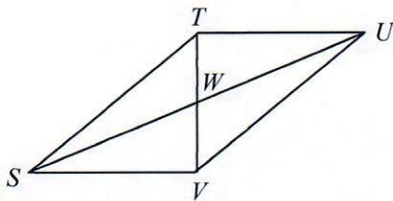
7. Solve for x .



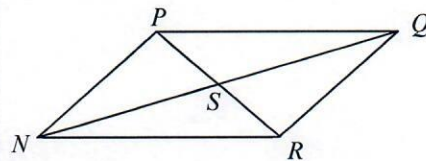
8. Find YZ .



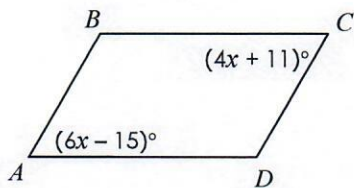
9. If $TV = 74$ and $WV = 4x + 1$, solve for x .



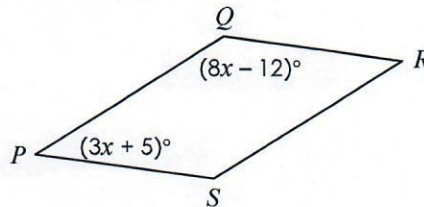
10. If $NS = 2x + 7$ and $SQ = 5x - 23$, find NQ .



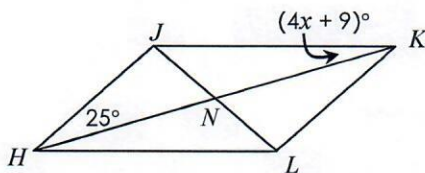
11. Find $m\angle B$.



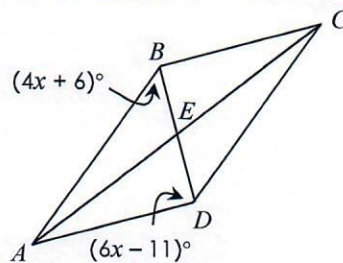
12. Find $m\angle R$.



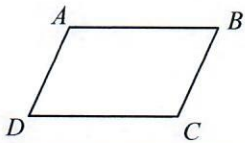
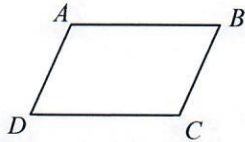
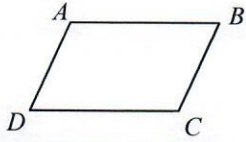
13. If $m\angle KLH = 134^\circ$, solve for x .



14. If $m\angle ABC = 115^\circ$, find $m\angle ADB$.



PROVING PARALLELOGRAMS *in the Coordinate Plane*

METHOD 1	<p>Prove both pairs of opposite sides are congruent.</p>  <p>If _____ and _____, then $ABCD$ is a parallelogram.</p>	Use....
METHOD 2	<p>Prove both pairs of opposite sides are parallel.</p>  <p>If _____ and _____, then $ABCD$ is a parallelogram.</p>	Use....
METHOD 3	<p>Prove one pair of opposite sides are congruent and parallel.</p>  <p>If _____ and _____, then $ABCD$ is a parallelogram.</p>	Use....

SET Use the **distance formula** to determine if the figure is a parallelogram.

1. $A(-7, 4)$, $B(1, 2)$, $C(9, -8)$, $D(1, -6)$

2. $P(-4, 2)$, $Q(6, 4)$, $R(11, -2)$, $S(2, -3)$

SET 2 Use the **slope formula** to determine if the figure is a parallelogram.

3. $W(-7, -4)$, $X(1, -6)$, $Y(5, -13)$, $Z(1, -12)$

4. $E(0, 8)$, $F(6, 10)$, $G(2, 0)$, $H(-4, -2)$

SET 3 Use the **distance formula AND slope formula** to determine if the figure is a parallelogram.

5. $J(-9, -2)$, $K(-5, 1)$, $L(1, -4)$, $M(-3, -7)$

6. $S(1, 5)$, $T(10, 7)$, $U(14, 1)$, $V(-3, -1)$

Name: _____

Unit 7: Polygons & Quadrilaterals

Date: _____ Per: _____

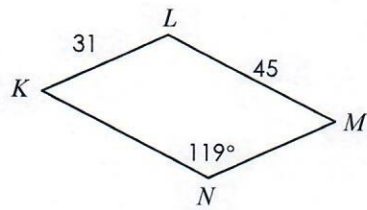
Homework 2: Parallelograms



** This is a 2-page document! **

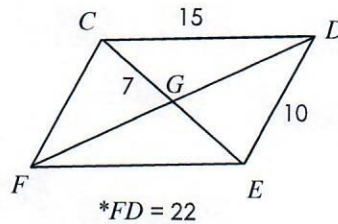
Directions: If each quadrilateral below is a parallelogram, find the missing measures.

1.



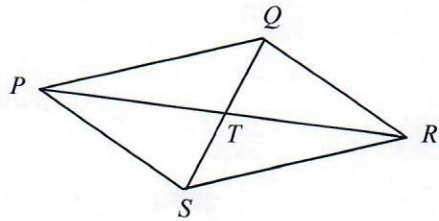
- $MN =$ _____
- $KN =$ _____
- $m\angle K =$ _____
- $m\angle L =$ _____
- $m\angle M =$ _____

2.



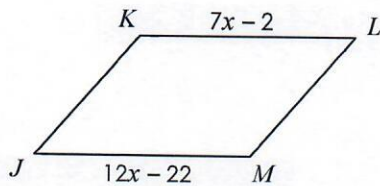
- $CF =$ _____
- $FE =$ _____
- $CE =$ _____
- $GD =$ _____

3. Given $PQ = 24$, $PS = 19$, $PR = 42$, $TQ = 10$, $m\angle PQR = 106^\circ$, $m\angle QSR = 49^\circ$, and $m\angle PRS = 35^\circ$.

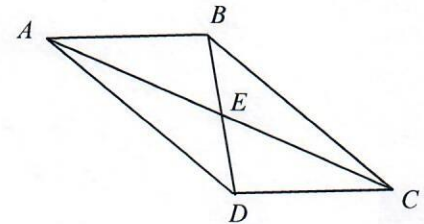


- $QR =$ _____
- $SR =$ _____
- $PT =$ _____
- $SQ =$ _____
- $m\angle QRS =$ _____
- $m\angle PQS =$ _____
- $m\angle RPS =$ _____
- $m\angle PSQ =$ _____

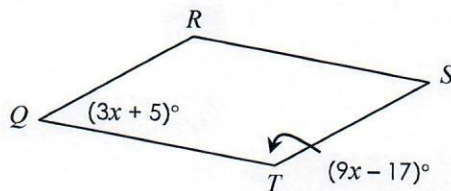
4. Find KL .



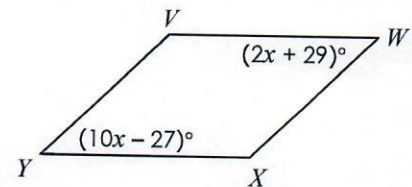
5. If $AC = 8x - 14$ and $EC = 2x + 11$, solve for x .



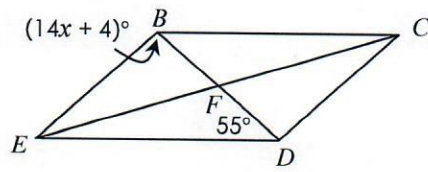
6. Solve for x .



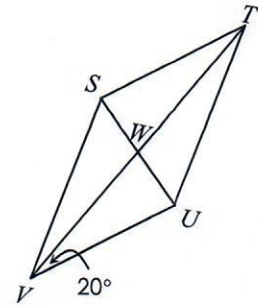
7. Find $m\angle V$.



8. If $m\angle BCD = 51^\circ$, solve for x .



9. If $m\angle VST = (5x + 23)^\circ$ and $m\angle VUT = (8x - 49)^\circ$, find $m\angle SVT$.



Directions: Determine whether the quadrilateral is a parallelogram using the indicated method.

10. $Q(-10, -2), R(1, -1), S(1, -7), T(-11, -8)$ (Distance Formula)

11. $K(2, 7), L(6, 12), M(13, 13), N(9, 8)$ (Slope Formula)

12. $D(-5, -6), E(5, 2), F(4, -4), G(-6, -12)$ (Distance & Slope Formulas)