Algebra 2, Period 9 – Dr Sargent – Work for April 13-24

April 13-17 – Arithmetic Operations involving Polynomials

We are now beginning o new unit on Polynomial Functions. We begin with a definition of a monomial, and then of a polynomial. On the attached worksheet, you will notice that the definition of a monomial is a number, variable, or a product of numbers and variables. In other words, it is a term and cannot have any addition or subtraction. Examples might be: 3, 5/4, 5x, 3i, π, or $6x^3$. All of these are monomials. A polynomial, then, is simply sum or difference of more than one monomial. In other words, a polynomial is a string of monomials tied together with plusses and minuses.

You can simplify monomials by using the exponent rules, as follows:

Name	**Rule**	**Example**
Product Rule	$a^m \cdot a^n = a^{m+n}$	$x^2 \cdot x^7 = x^9$
Power Rule	$(a^m)^n = a^{m \cdot n}$	$(x^2)^7 = x^{14}$
Quotient Rule	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^7}{x^2} = x^5$
Negative Exponent Rule	$a^{-m} = \frac{1}{a^m}$	$x^{-2} = \frac{1}{x^2}$
Zero Exponent Rule	$a^0 = 1$(unless a = 0, then it is undefined)	$(7x^2)^0 = 1$

With these rules in mind, let's see if we can simplify some monomials. Look at Example 1 on the attached sheet: $5x^2 \cdot (-7)x^6$ We begin by combining like terms like this: $(5 \cdot (-7))(x^2 \cdot x^6)$. Now we multiply the like terms and get $-35x^8$. Now you try problem #2.

Look now at Example #3: $\frac{54m^6n^4}{3m^2n} - 10m^4n^3$. At first glance, this seems very difficult, but if we take it step by step, it is not so bad. Begin by dividing like terms: $\frac{54}{3} = 18$; $\frac{m^6}{m^2} = m^4$; *and* $\frac{n^4}{n} = n^3$. Now the problem looks like this: $18m^4n^3 - 10m^4n^3$. Since these are now like terms, we simply subtract the coefficients and end up with $8m^4n^3$ as our answer. Now, you try the rest of the example. I will give you the answers at the end of these instructions, but please try them yourself first.

A polynomial, as we said, is a string of monomials. Each monomial in the polynomial is called a term. For an expression to be a polynomial term, any variables in the expression must have whole-number powers (or else the "understood" power of 1, as in x^1, which is normally written as x). A plain number can also be a polynomial term. In particular, for an expression to be a polynomial term, it must contain no square roots of variables, no fractional or negative powers on the variables, and no variables in the denominators of any fractions. Here are some examples:

This is NOT a polynomial term: $6x^{-2}$, because there cannot be negative powers of a variable in a polynomial term.

This IS a polynomial term: $4x^2$

This is NOT a polynomial term: $\frac{1}{x^2}$, because the variable cannot be in the denominator

This IS a polynomial term: $\frac{5^{\frac{2}{3}}}{4}\sqrt{3}x^{47}$, because the variable has an even-numbered exponent.

So, here are examples of polynomials:

$4x^2 - 5x + 3$

$2x^5 + 3y - z$

We say that the highest exponent in the polynomial is the degree of the polynomial. In the first example above, the degree would be 2; in the second example of a polynomial, the degree would be 5.

When you write a polynomial in standard form, you write it with the exponents in descending order. Both the examples above are already in standard form. Now, you try doing problems 10-12 on the worksheet. These are somewhat difficult, but keep two rules in mind: 1) List the terms in the order of descending exponents and 2) where there is more than one variable with the same exponent, list them alphabetically in descending order of exponents. Always, starting with the variable that has the highest exponent, and list that one in descending order before moving on to the next variable.

We classify polynomials based on the degree of the highest exponent, and on the number of terms in the polynomial, like this:

Degree	**Classification**
0	Constant
1	Linear
2	Quadratic
3	Cubic
4	Quartic
5	Quintic

Number of Terms	**Classification**
1	Monomial
2	Binomial
3	Trinomial
4	Polynomial

Looking at the examples on the worksheet, Example 13 would be classified as: linear, binomial. See if you can complete the worksheet. You may now check your answers.

The next worksheet deals with doing arithmetic operations on polynomials. When you add or subtract polynomials, you simply add or subtract like terms. One thing you will need to remember is when subtracting, make sure you distribute the negative sign to everything inside the following parenthesis. Look, for example, at Problem #3: $(5k^3 - 2k^2 + 2k) - (2k^2 + 2k + 17)$. To solve this problem, remember that everything inside the second parenthesis will be affected by the subtraction sign preceding the parenthesis, so every sign inside the parenthesis changes to its opposite before you can combine like terms. Your answer would then be: $5k^3 - 4k^2 + 17$. Now you try problems 1-5 and see how you do.

When you multiply polynomials, the key is to distribute carefully, so that every term in one parenthesis is multiplied by every term in the other. So, for example, take Problem 6: $(w - 7)(w^2 + 2w + 1)$. Every term in the second parenthesis must be multiplied by every term in the first. We distribute like this: $w(w^2 + 2w + 1) - 7(w^2 + 2w + 1)$. Now when we multiply, we get: $w^3 + 2w^2 + w - 7w^2 - 14w - 7$. Last, we combine like terms to obtain our final answer: $w^3 - 5w^2 - 13w - 7$. Now you try problems 7-12/ Once you have tried, you can check your answers at the end of this instruction sheet. How did you do?

Finally, we deal with how we divide a polynomial by a monomial. In essence, you divide each term in the polynomial by the monomial. So, for example, Problem 13 is: $\frac{9m^3-24m^2}{3m}$. You divide each term in the numerator by the denominator, so it goes like this: $\frac{9m^3}{3m} - \frac{24m^2}{3m} = 3m^2 - 8m$. Notice that you divide the numbers by the numbers and the variables by the variables; that is, the 9 and the 24 in the numerator are divided by the 3 in the denominator; and the $m^3$and the m^2in the numerator are divided by the m in the denominator. Now you try the balance of the problems on this page. How did you do?

Once you have a good grasp of this, please complete homework 5-1 on ALEKS. This will be available through Sunday, April 26. If you need additional help, I would recommend your viewing the Khan Academy videos available here: https://www.khanacademy.org/math/algebra-home/alg-polynomials

If you still need help after viewing the videos, please do not hesitate to contact me and I will try to help you with whatever you are struggling with.

Continuing ALEKS assignment Please remember that your regular weekly assignment of 12 topics on ALEKS continues.

Answers to Selected Questions

Worksheet on Monomials:

Example 2: $32a^7b^{11}$

Example 4: $\frac{20}{k^3}$ (Remember, a negative exponent moves the variable across the fraction bar line and the exponent then becomes positive after the variable is moved).

Example 5: $\frac{2}{27}r^7s^{15}$

Example 6: $\frac{1}{2w^9}$

Example 7: $\frac{5y}{8x^2}$

Example 8: 81

Example 9: Answers will vary

Example 10: $-k^5 - 3k^3 + \frac{1}{4}k^2 + 8k - 1$

Example 11: $4a^3 + 18a^2b^2 + 7ab - b^2$

Example 12: $9x^3y - x^2 + 5xy^2 - y^4 + 2$

Example 14: quintic, trinomial

Example 15: constant, monomial

Example 16: cubic, polynomial

Example 17: quadratic, trinomial

Example 18: quartic, monomial

Examples 19 & 20: Answers will vary

Adding and Subtracting Polynomials Worksheet

Problem 1: $4x^2 + 6x - 9$

Problem 2: $9n^3 - 6n^2 + 3n - 3$

Problem 4: $y^3 + 2y^2 + 6y - 4$

Problem 5: $-9a^2 + 11ab + 9b^2$

Problem 7: $-24x^2 - 23x + 4$

Problem 8: $-3a^3 - 33a^2 - 42a + 240$

Problem 9: $8c^3 + 60c^2d + 150cd^2 + 125d^3$

Problem 10: $6m^4 + 7m^3 - 45m^2 + 41m - 9$

Problem 11: $-x^4 - 15x^3 - 51x^2 + 39x + 28$

Problem 12: $A = 4x^2 + 16x - 9$

Problem 14: $5x^2 - x - 2$

Problem 15: $7a^3b^2 - 10ab + 1$

Problem 16: $\frac{5c^6d^2}{2} - 3c^4d + \frac{c}{2}$

Problem 17: $10x - 4 + \frac{3}{x}$

Problem 18: $\frac{5m}{n} - 3n^2$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

MONOMIALS

- A **monomial** is a number, variable, or a product of numbers and variables.
- Examples: _____
- Use the **EXPONENT RULES** to simplify monomial expressions:

NAME	RULE	EXAMPLE
Product Rule		
Power Rule		
Quotient Rule		
Negative Exponent Rule		
Zero Exponent Rule		

When **ADDING OR SUBTRACTING** monomials, **COMBINE LIKE TERMS!**

EXAMPLES

1. $5x^2 \cdot -7x^6$

2. $(-2a^3b)^2 \cdot 8ab^9$

3. $\frac{54m^6n^4}{3m^2n} - 10m^4n^3$

4. $2k^4 \cdot 10k^{-7}$

5. $\left(\frac{2}{3}r^2s^7\right)^2 \cdot \left(\frac{1}{6}r^3s\right)$

6. $\left(\frac{14w^{12}}{7w^3}\right)^{-1}$

7. $\frac{15x^{10}y^4}{24x^{12}y^3}$

8. $\left(\frac{c}{c^2}\right)^4 \cdot (-3c)^4$

9. Give an example of two monomials with a quotient of $\frac{-3n^2}{m}$

POLYNOMIALS

- A **polynomial** is the sum or difference of many monomials.
- The **highest exponent** of a polynomial is called the _____.
- **Standard Form:** _____

_____

Write the polynomials below in standard form:

10. $-k^5 - 1 + 8k - 3k^3 + \frac{1}{4}k^2$ _____

11. $18a^2b^2 + 7ab - b^2 + 4a^3$ _____

12. $5xy^2 - x^2 + 9x^3y - y^4 + 2$ _____

CLASSIFYING POLYNOMIALS

Degree	
0	
1	
2	
3	
4	
5	

Number of Terms	
1	
2	
3	
4+	

Polynomials are **classifed by degree** (highest exponent) and **number of terms**. Use the charts to the left to classify each polynomial below.

13. $-3x + 1$ _____

14. $9x^5 - x^4 + 2x$ _____

15. 24 _____

16. $\frac{1}{2}x^3 - 2x^2 + 4x + 15$ _____

17. $-x^2 - 18x + 31$ _____

18. $-\frac{3}{2}x^4$ _____

19. Give an example of a cubic binomial. _____

20. Give an example of a linear monomial. _____

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

Adding & Subtracting Polynomials

① **COMBINE LIKE TERMS!** (Watch out for subtraction problems!)

② Write your answer in **STANDARD FORM.**

1. $(3x^2+11x+4)+(-5x+x^2-13)$

2. $(9n^3-4n^2+2n-10)+(-2n^2+n+7)$

3. $(5k^3-2k^2+2k)-(2k^2+2k+17)$

4. $(y+4y^2-3)-(1+2y^2-5y-y^3)$

5. Subtract $(-10ab+7a^2-b^2)$ from $(8b^2+ab-2a^2)$.

Multiplying Polynomials

① **DISTRIBUTE** or **FOIL.**

② **COMBINE LIKE TERMS!**

③ Write your answer in **STANDARD FORM.**

6. $(w-7)(w^2+2w+1)$

7. $(2x+1)(4-9x)-2x(3x+11)$

8. $-3(a+5)(a-2)(a+8)$

9. $(2c+5d)^3$

10. $(3m^2 - 4m + 1)(2m^2 + 5m - 9)$

11. $(x^2 + 6x - 7)(-x^2 - 9x - 4)$

12. The height of a trapezoid can expressed as $(2x + 9)$ while the bases can be expressed as $(3x - 7)$ and $(x + 5)$. Write an expression to represent the area of the trapezoid.

Dividing Polynomials (by a Monomial)

1. **DIVIDE** each term in the numerator by the monomial in the denominator.
2. Write your answer in **STANDARD FORM.** Rewrite all terms with negative exponents.

13. $\dfrac{9m^3 - 24m^2}{3m}$

14. $\dfrac{20x^3 - 4x^2 - 8x}{4x}$

15. $(14a^4b^3 - 20a^2b^2 + 2ab) \cdot (2ab)^{-1}$

16. $\dfrac{30c^8d^3 - 36c^6d^2 + 6c^3d}{12c^2d}$

17. $\dfrac{30x^3 - 12x^2 + 9x}{3x^2}$

18. $\dfrac{40m^3n^4 - 24m^2n^7}{8m^2n^5}$