Honors Geometry – May 11-29 - Dr. Sargent – Periods 4 & 8

Week of May 11-15 – Inscribed Angles and Inscribed Quadrilaterals

This week, we continue learning about circles and, in particular, about inscribed angles and inscribed quadrilaterals. On the attached worksheet, an inscribed angle is defined as an angle with its vertex on the circle with two sides that are chords. In the illustration on the worksheet, $\angle ABC$ is an inscribed angle. An intercepted arc is the arc that lies between the endpoints of an inscribed angle – in the illustration, it is $\widehat{AC}$. The measure of the inscribed angle is equal to half the measure of the intercepted arc. In the illustration on the worksheet, $m\angle ABC = \frac{1}{2} m\,\widehat{AC}$.

If an inscribed angle intercepts a diameter, then it is a right angle, which makes sense because it would be ½ of 180°. On the illustration on the worksheet, $\angle BAC = 90°$. You can see that $\angle BAC$ intercepts the circle at the endpoints of the diameter $\overline{BC}$, and thus$\angle BAC = \frac{1}{2}\ 180° = 90°$.

Looking at the third illustration, it should be apparent that if two inscribed angles intercept the same arc, then the angles are congruent. In other words, $\angle ABD \cong \angle ACD$. (or $m\angle ABD = m\angle ACD$).

So, let's look at Problem 1: $\angle WXY$ intercepts $\widehat{WY}$. Since $\widehat{WY} = 62°$, then $m\angle WXY = \frac{1}{2} m\,\widehat{WY} = \frac{1}{2} \cdot 62° = 31°$. Problem 2 is simply the same type of problem in reverse: $m\angle DEF = 113°$. The intercepted arc DF must therefore be twice the measure of the intercepted arc, so $m\,\widehat{DGF} = 2 \cdot 113 = 226°$.

Can you see in Problem 3 that $\angle PQR$ intercepts a diameter and, thus, must be a right angle? Therefore, $m\angle PQR = 90°$. See if you can now do problems 4-6 (Answers at the end of these instructions – but please try them yourself before looking at the answers).

Beginning with problem 7, we simply begin substituting an algebraic expression for a measure and then solving for the variable. Beginning with Problem 13, we begin substituting the value of the variable back to find the measure of the arc or angle for which we are solving. For example, look at Problem 13. We are told that $m\angle FGH = 6x + 21; and\ m\,\widehat{FJH} = 17x - 28$. Since the measure of the arc must be double the measure of the angle, we can write the problem this way: $2(6x + 21) = 17x - 28$ When we solve for x, we discover $x = 14$. Now we substitute this 14 in place of x in the measure of the arc: $m\,\widehat{FJH} = 17(14) - 28 = 210°$.

See if you can solve the remainder of the problems on the worksheet. You will NOT need to return this worksheet – it is to help you in your understanding of the relationship between the measures of inscribed angles and intercepted arcs.

Inscribed Quadrilaterals – The next worksheet is about Inscribed Quadrilaterals. The main idea is that if a quadrilateral is inscribed in a circle, then its opposite angles are supplementary. When we say an angle or a polygon is inscribed, we mean that the end points of all the sides (the vertices) lie on the circumference of the circle; that is to say, all the sides are chords. In the illustration on the worksheet, $m\angle A + m\angle C = 180°$; and $m\angle B + m\angle D = 180°$.

We can now begin using this knowledge to find missing angles in inscribed quadrilaterals. In problem 1: angles J & L are supplementary, as are angles K & M. thus we can find the missing angles this way: $m\angle J + m\angle L = 180; m\angle J + 92 = 180; m\angle J = 88°$. Similarly, $m\angle K + m\angle M = 180; m\angle K + 45 = 180; m\angle K = 135°$.

In problem 2, we must first find $m\angle R$ before we can find $m\angle P$. We know that $m\angle QRS = \frac{1}{2} m\,\widehat{QPS} = \frac{1}{2}(126 + 90) = 108°$. Since $\angle P$ is supplementary to $\angle R$, then $m\angle P = 180 - 108 = 72°$. Then $m\angle S + m\angle Q = 180; m\angle S + 93 = 180; m\angle S = 87°$. Now see if you can solve the balance of the problems on this worksheet. Once again, this does NOT need to be returned to me – it is for your practice.

I have included a sheet called "The Giant Circle Challenge" which you may complete and return to the school by May 29 for extra credit. For those doing this online, you may simply message me the answers – make sure they are clearly labeled as "The Great Circle Challenge" and each answer is properly labelled. Of course, you can scan the page or attach a picture of the page if you wish.

I have attached Unit 10 Homework 5 for you to complete and return to me, preferably by May 22, but definitely by May 29. You may return a written copy to school, or you may message me the answers, a scan, or a picture of the page.

Week of May 18-22 – Tangent Lines

The last thing we will apparently have time for this year is a lesson on Tangent Lines and circles. I have attached a worksheet Tangent Lines which I ask you to now take a look at. A tangent, you will recall, is a line that intersects a circle at exactly one point, called the point of tangency. A line is tangent to a circle if and only if it is perpendicular to a radius drawn to the point of tangency. In the illustration to the left, $\overleftrightarrow{AB}$ *is a tangent, and is perpendicular to Radius PA.*

Let's now begin to apply this knowledge to problems of a circle. Take a look at problem 1: We know that line segment AB will be tangent to Circle P if and only if radius PA is perpendicular to AB. We can determine this using the Pythagorean Theorem to see if triangle APB is a right triangle: $8^2 + 15^2 = 17^2; 64 + 225 = 289; 289 = 289$. Thus, $\angle PAB = 90°$, and AB is a tangent line to Circle P. You can now try Problem 2 to see if AB is a tangent line.

Beginning in Problem 3, we are given that we have a tangent line, and we are to solve for a missing measure. We can use the Pythagorean Theorem and solve for the missing measure. Problem 3 would look like this: $7^2 + 19^2 = x^2; 410 = x^2; x = \sqrt{410} \approx 20.2$. Now you try the remainder of problems4-9, rounding to the nearest tenth.

Following problem 9, you are given two more properties of Tangent Lines. The first is that if two segments from the same external point are tangent to a circle, then they are congruent. This can be easily proven on the given illustration by drawing a line from the center of the circle, D, to the common external point, A. We would then have two right triangles with a common hypotenuse (DA). Using HL, we can prove the triangles are congruent because the radii (AD & CD) are congruent. Thus, using CPCTC, legs AB and BC must also be congruent.

The final property given to us is that if a polygon is circumscribed around a circle, then all sides are tangent. To circumscribe simply means to draw one figure around another so that each side of the outside figure just touches the inner figure without cutting the inner figure. In the illustration given, the sides of the circumscribed triangle are tangent to the inner circle at points D, E and F.

See if you can now solve problems 9-14, remembering the properties we have just been learning. These problems are worked for you at the end, but please try them yourself before referring to the answers.

I have enclosed Unit 10 Homework 6 for you to complete and return. It should be returned by May 29 either in written form to the school, or you can message me the answers, a scan, or a picture of the completed page.

May 25-29 – Final Exam

I have placed a final exam on ALEKS that you should complete by May 31. If you are doing it in written form, it will be attached, and you should return it to the school by May 29. If you are doing the written form, I have attached two copies: One copy which you may do as a practice with the answers attached so you can check yourself, and one copy which will need to be returned for grading. If you are doing this online, you will have 3 chances, so you can use the first time through as a review, and then go back and do the test for real. Once you begin the test, do not leave the test or ALEKS will think you are finished.

Ongoing ALEKS assignment

Do not forget that you have the regular weekly 12 topics assigned for ALEKS through May 31.

Answers to Selected Problems:

Inscribed Angles Worksheet

Problem 4: $m\,\widehat{BC} = 86°$

Problem 5: $m\angle JKL = 121°$

Problem 6: $m\angle RST = 41°; m\angle RUT = 41°$

Problem 7: $8x - 9 = 79; x = 11$

Problem 9: $13x - 1 = 90; x = 7$

Problem 11: $7x - 10 = 53; x = 9$

Problem 14: $2(5x - 16) = 12x - 50; 10x - 32 = 12x - 50;\ -32 = 2x - 50; 18 - 2x; x = 9; m\angle STU = 5(9) - 16 = 29°$

Problem 16: $10x - 30 = 90; x = 12; m\angle KJL = 3(12) + 2 = 38°; m\,\widehat{KL} = 2 \cdot 38 = 76°$

Inscribed Quadrilaterals

Problem 3: $m\angle A = 104°; m\angle B = 130°; m\angle C = 76°; m\angle D = 50°\text{C}$

Problem 4: $9x - 5 + 68 = 180; x = 13$

Problem 6: $15x - 23 + 113 = 180; x = 6$

Problem 8: $5x + 1 + 13x - 37 = 180; x = 12; m\angle Y = 13(12) - 37 = 119°$

Homework 5

Problem 1: $m\,\widehat{JML} = 176°$

Problem 4: $m\,\widehat{RS} = 105°$

Problem 7: $m\angle A = 79°; m\angle B = 112°$

Problem 10: $x = 2$

Problem 13: $x = 12$

Problem 16: $m\widehat{ST} = 125°$

Tangent Lines Worksheet

Problem 2: $11^2 + 14^2 = 16^2; 317 \neq 256$; therefore AB is NOT a tangent line to Circle P

Problem 4: $11^2 + x^2 = 18^2;\ x^2 = 203; x \approx 14.2$

Problem 6: $22^2 + x^2 = 25^2;\ x^2 = 141; x \approx 11.9$

Problem 8: $Let\ y = the\ diameter;\ y^2 + 7^2 = 25^2;\ y^2 = 576; y = 24; x = \frac{1}{2}(24) = 12$

Problem 10: $8x - 19 = 5x + 23; x = 14$

Problem 12: $10x - 41 = 4x + 7; x = 8; ST = 10(8) - 41 = 39; Let\ y = SV;\ y^2 = 39^2 + 17^2;\ y^2 = 810; y \approx 42.5$

Problem 14: $Perimeter = 180$

Homework 6

Problem 1: Yes

Problem 3: No

Problem 5: $x \approx 16.6$

Problem 8: $x \approx 28.3$

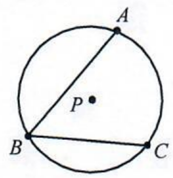
Problem 11: $x = 9$

Problem 14: $FD \approx 25.9$

Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

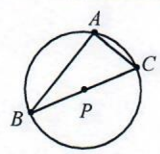
INSCRIBED ANGLES



$m\angle ABC =$ ____________

- An **inscribed angle** is an angle with its vertex ______ the circle with two sides that are ______________.
- An **intercepted arc** is the arc that lies between the ______________ of an inscribed angle.
- The **measure of the inscribed angle** is equal to ________ the measure of its **intercepted arc.**

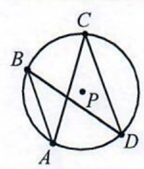
INTERCEPTING a Diameter



If an inscribed angle **intercepts a diameter**, then it is a ____________ angle.

$m\angle BAC =$ ____________

OVERLAPPING Arcs

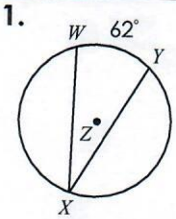


If two inscribed angles intercept the **same arc**, then the angles are ______________.

$m\angle ABD =$ ____________

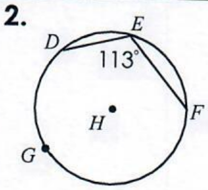
Directions: Find each angle and arc measures.

1.



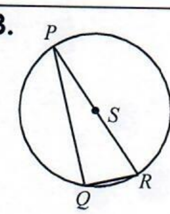
$m\angle WXY =$ ________

2.



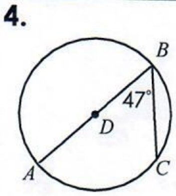
$m\widehat{DGF} =$ ________

3.



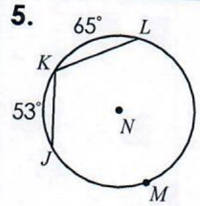
$m\angle PQR =$ ________

4.



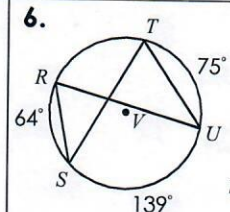
$m\widehat{BC} =$ ________

5.



$m\angle JKL =$ ________

6.

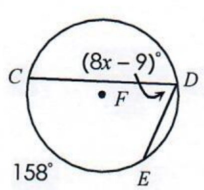


$m\angle RST =$ ________

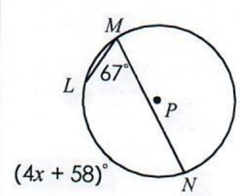
$m\angle RUT =$ ________

Directions: Find each value or measure.

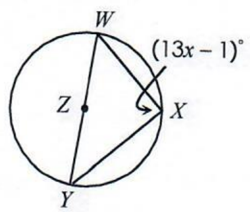
7. Solve for x.



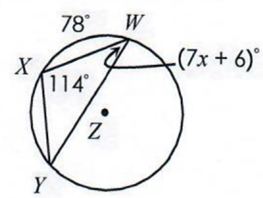
8. Solve for x.



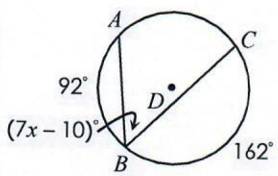
9. Solve for x.



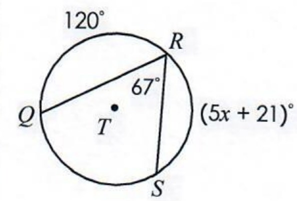
10. Solve for x.



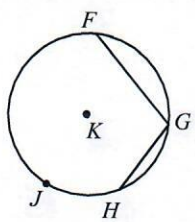
11. Solve for x.



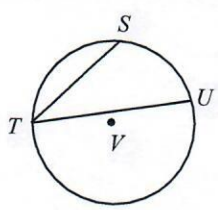
12. Solve for x.



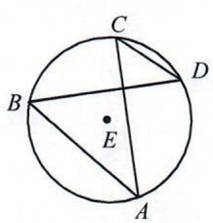
13. If $m\angle FGH = (6x + 21)^\circ$ and $m\overset{\frown}{FJH} = (17x - 28)^\circ$, find $m\overset{\frown}{FJH}$.



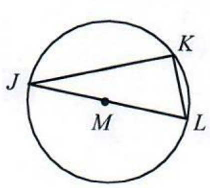
14. If $m\angle STU = (5x - 16)^\circ$ and $m\overset{\frown}{SU} = (12x - 50)^\circ$, find $m\angle STU$.



15. If $m\angle ABD = (6x + 26)^\circ$ and $m\angle ACD = (13x - 9)^\circ$, find $m\overset{\frown}{AD}$.

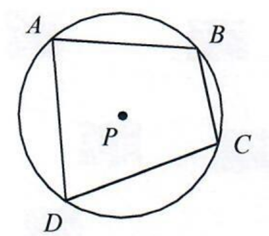


16. If $m\angle KJL = (3x + 2)^\circ$ and $m\angle KLJ = (7x - 32)^\circ$, find $m\overset{\frown}{KL}$.



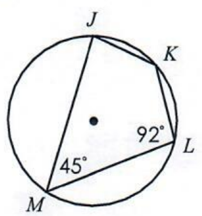
Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples
Inscribed **QUADRILATERALS**	If a **quadrilateral is inscribed in a circle**, then its **opposite angles** are ______________________. In circle P to the right: ______________________ and ______________________



Directions: Find each value or measure.

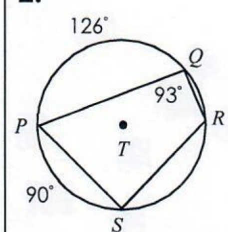
1.



$m\angle J =$ ______

$m\angle K =$ ______

2.

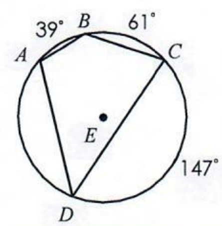


$m\angle P =$ ______

$m\angle R =$ ______

$m\angle S =$ ______

3.



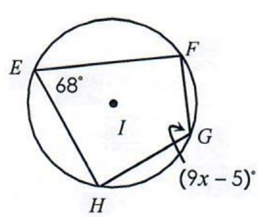
$m\angle A =$ ______

$m\angle B =$ ______

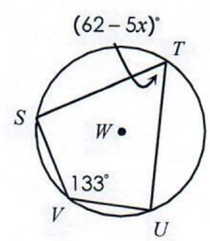
$m\angle C =$ ______

$m\angle D =$ ______

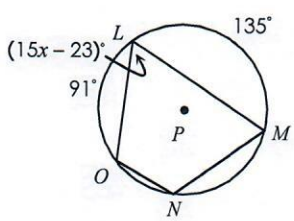
4. Solve for x.



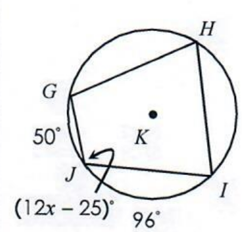
5. Solve for x.



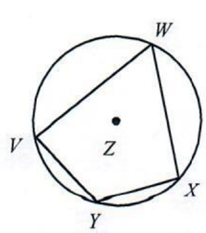
6. Solve for x.



7. Solve for x.



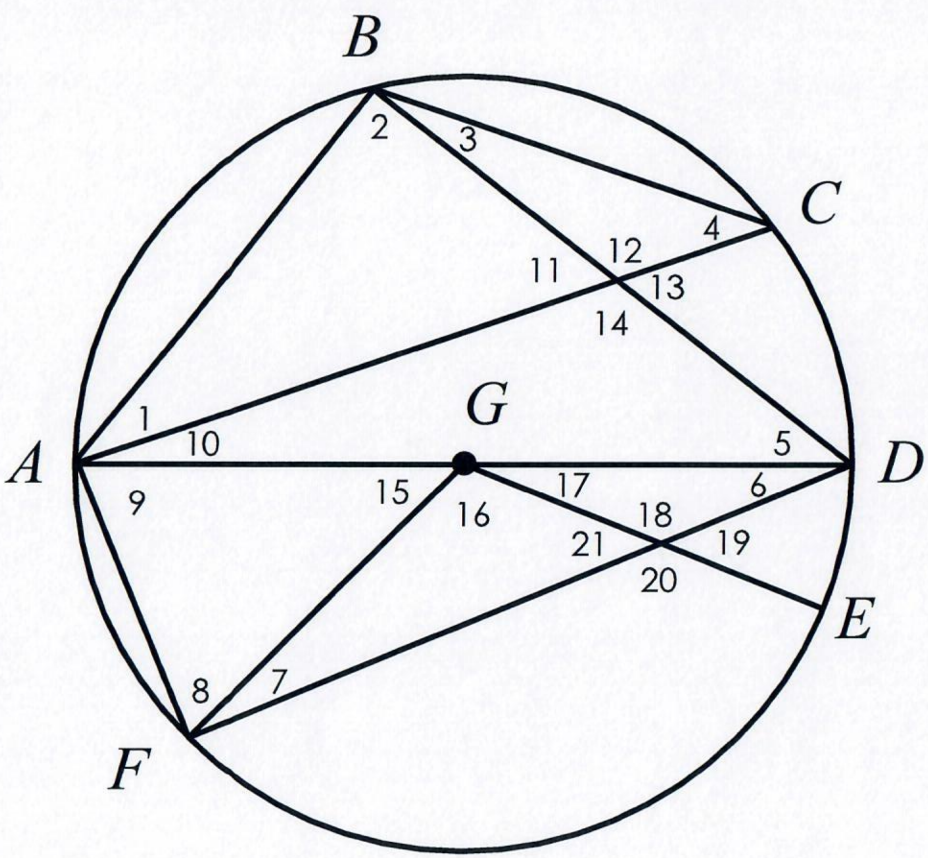
8. If $m\angle W = (5x + 1)^\circ$ and $m\angle Y = (13x - 37)^\circ$, find $m\angle Y$.



The Giant CIRCLE CHALLENGE!

Name: ______________________

Date: ______________ Period: ______



Given: G is the center of the circle
$\overline{AD}$ is a diameter, $m\overset{\frown}{AB} = 78^\circ$,
$m\overset{\frown}{FE} = 105^\circ$, $m\overset{\frown}{ED} = 27^\circ$, $m\overset{\frown}{CD} = 42^\circ$

Find each angle measure:

$m\angle 1$ = ______

$m\angle 2$ = ______

$m\angle 3$ = ______

$m\angle 4$ = ______

$m\angle 5$ = ______

$m\angle 6$ = ______

$m\angle 7$ = ______

$m\angle 8$ = ______

$m\angle 9$ = ______

$m\angle 10$ = ______

$m\angle 11$ = ______

$m\angle 12$ = ______

$m\angle 13$ = ______

$m\angle 14$ = ______

$m\angle 15$ = ______

$m\angle 16$ = ______

$m\angle 17$ = ______

$m\angle 18$ = ______

$m\angle 19$ = ______

$m\angle 20$ = ______

$m\angle 21$ = ______

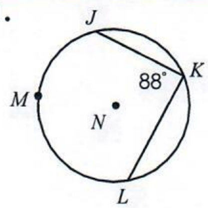
Name: ____________________ **Unit 10:** Circles

Date: ____________________ **Per:** ________ **Homework 5:** Inscribed Angles

**** This is a 2-page document! ****

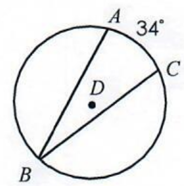
Directions: Find each angle or arc measure.

1.



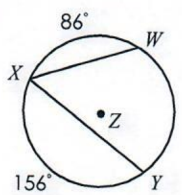
$m\widehat{JML}$ = ________

2.



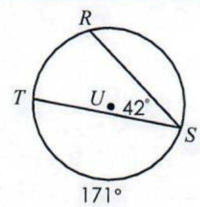
$m\angle ABC$ = ________

3.



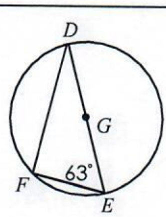
$m\angle WXY$ = ________

4.



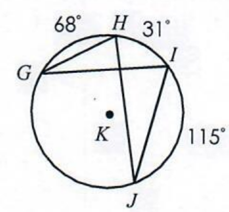
$m\widehat{RS}$ = ________

5.



$m\widehat{FE}$ = ________

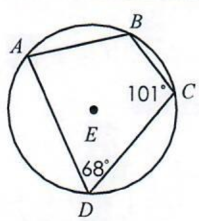
6.



$m\angle GHJ$ = ________

$m\angle GIJ$ = ________

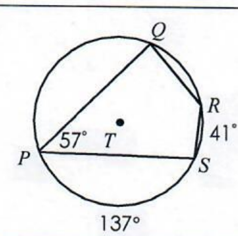
7.



$m\angle A$ = ________

$m\angle B$ = ________

8.



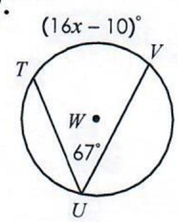
$m\angle Q$ = ________

$m\angle R$ = ________

$m\angle S$ = ________

Directions: Find the value of x.

9.



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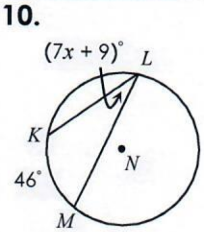

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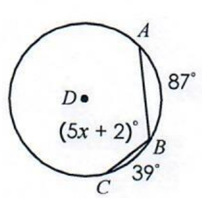

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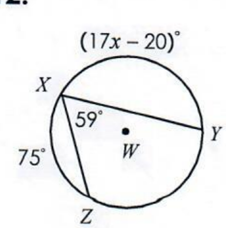

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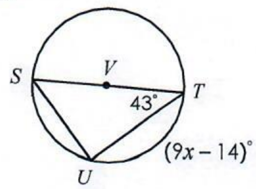

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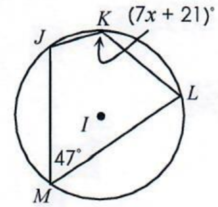
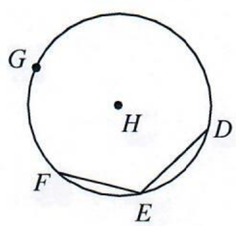
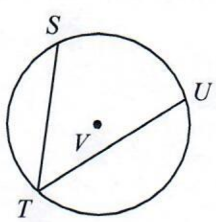


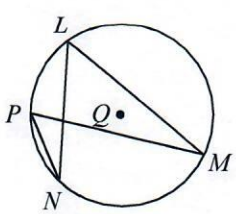
15. If $m\angle DEF = (6x + 37)^\circ$ and $mFGD = (19x - 31)^\circ$, find $m\angle DEF$.



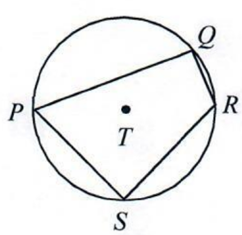
16. If $m\angle STU = (13x - 6)^\circ$, $mSU = (21x + 8)^\circ$, and $mUT = 143^\circ$, find mST.



17. If $m\angle LMP = (5x - 19)^\circ$ and $m\angle LNP = (2x + 11)^\circ$, find mPL.



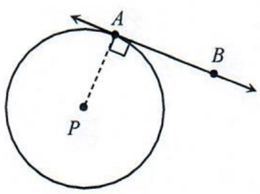
18. If $m\angle P = (4x - 1)^\circ$ and $m\angle R = (12x - 27)^\circ$, find $mQPS$.



Name:	Date:
Topic:	Class:

Main Ideas/Questions	Notes/Examples

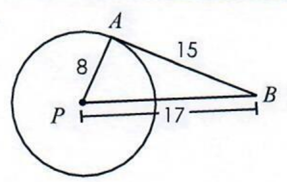
Tangent LINES



- A **tangent line** intersects a circle at exactly __________ point, called the **point of tangency.**
- A line is tangent to a circle if and only if it is ____________________ to a ____________ drawn to the point of tangency.

Directions: Determine if $\overline{AB}$ is tangent to circle P.

1.



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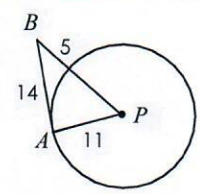
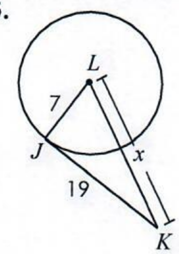


Directions: If $\overline{JK}$ is tangent to circle L, find x.

3.



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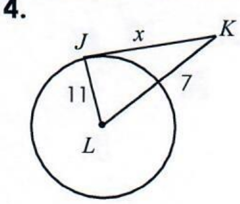

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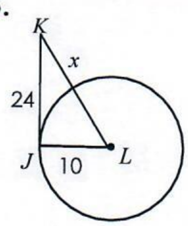
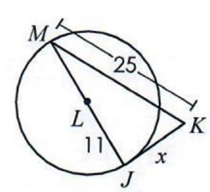


6.



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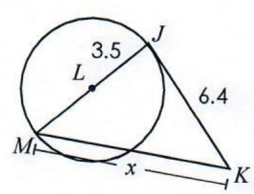

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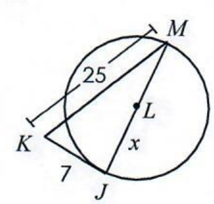

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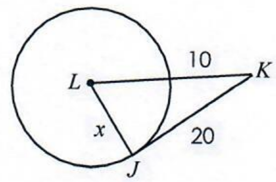
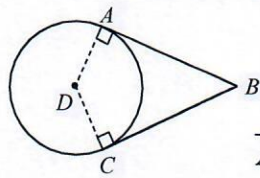


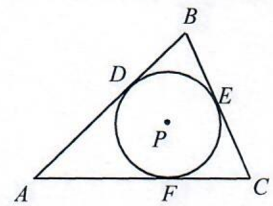
MORE TANGENT LINE Properties

If **two segments from the same external point** are **tangent** to a circle, then they are **congruent**.



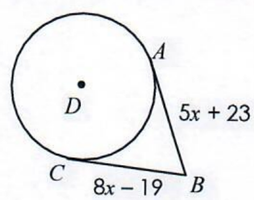
$\overline{AB} \cong \overline{CB}$

If a **polygon is circumscribed** around a circle, then **all sides are tangent**.

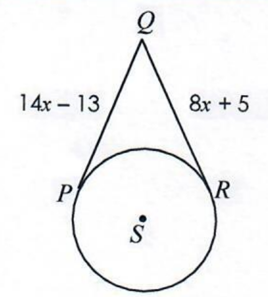


Directions: Find each value or measure. Assume that segments that appear to be tangent are tangent.

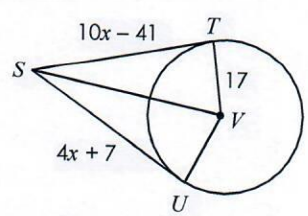
10. Find x.



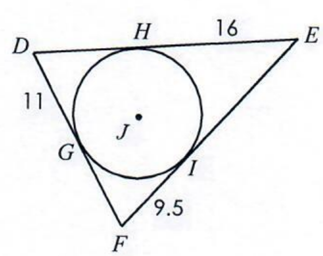
11. Find PQ.



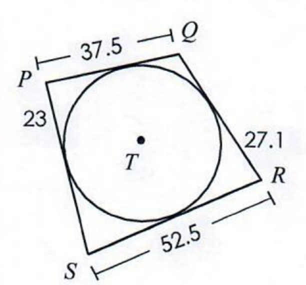
12. Find SV.



13. Find the perimeter of ΔDEF.



14. Find the perimeter of quadrilateral $PQRS$.



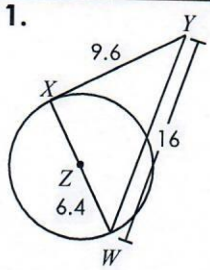
Name: ______________________________ **Unit 10:** Circles

Date: ____________________ **Per:** ______ **Homework 6:** Tangent Lines

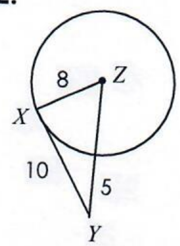
**** This is a 2-page document! ****

Determine if $\overline{XY}$ is tangent to circle Z.

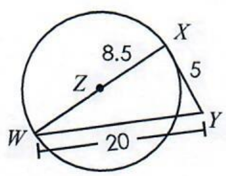
1.



2.



3.



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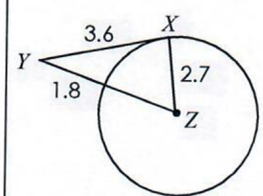
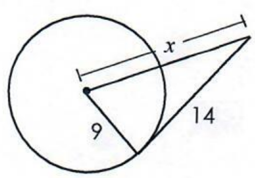


Find the value of x. Assume that segments that appear to be tangent are tangent.

5.



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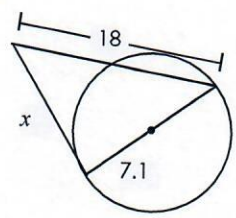

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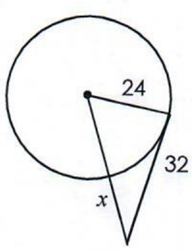

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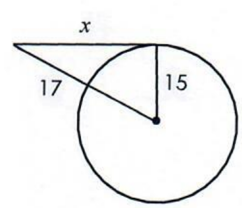

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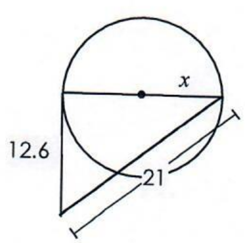
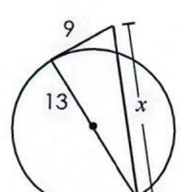
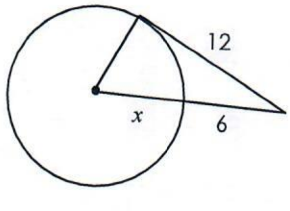


10.

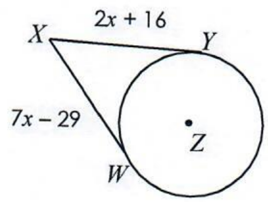


11.

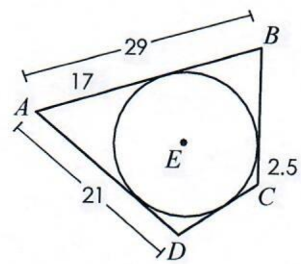


Find each value or measure. Assume that segments that appear to be tangent are tangent.

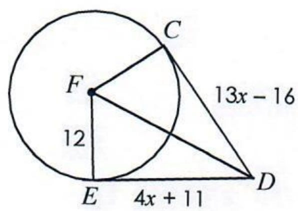
12. Find WX.



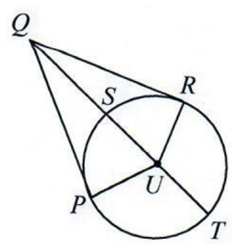
13. Find the perimeter of $ABCD$.



14. Find FD.



15. If $PQ = 4x + 2$, $QR = 7x - 19$, and $QU = 34$, find ST.



16. Find the perimeter of ΔJKL.