Mrs. Bledsoe 4th Period
Trig./Precalc.

Mrs. Bledsoe – Plans for Home Instruction for March 19 – 27, 2020

Trigonometry/Precalculus 4th Period:

18 Examples for trigonometric identities will be available in a separate file.

***Thursday, March 19, 2020:** Trigonometric Identities I WS Do # 2, 4, 5, 6.

***Friday, March 20, 2020:** Trigonometric Identities I WS Do # 9, 11, 16, 19.

Monday, March 23, 2020: Law of Sines Examples (pg.1 – 2), Do Law of Sines Pages 2-3 # 1 – 8

Tuesday, March 24, 2020: Law of Sines Pages 2-3 # 9-16

Wednesday, March 25, 2020: Law of Sines Pages 2-3 # 17 – 25

Thursday, March 26, 2020: Law of Sines Home Assignment WS (Day 6)

Friday, March 27, 2020: Law of Sines Home Assignment WS (Day 7)

Please do the best you can on these assignments. I hope you are not overwhelmed and that you do not get sick. Any messages via Livegrades or email are welcomed and encouraged. My school email is mbledsoe@k12.wv.us. I know this transition is a bit difficult and you may need help so I will be watching for questions and trying to help as much as possible.

Remember to take some breaks and relax after you have a plan on how to keep up with the work assigned and know how much break time you have. Don't procrastinate. Be responsible. Be safe.

Sincerely,

Mrs. Melissa Bledsoe

Trig/Precalc Mrs. Bledsoe 4th period

Days #1,2

Assignment: 3/19/2020 - Do #2, 4, 5, 6.
3/20/2020 - Do# 9, 11, 16, 19.

Write on your own paper

Prove each of the following identities.

1. $\tan x + \dfrac{\cos x}{1+\sin x} = \dfrac{1}{\cos x}$

3/19
$\boxed{2.}$ $\tan^2 x + 1 = \sec^2 x$

3. $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$

3/19
$\boxed{4.}$ $\tan x + \cot x = \sec x \csc x$

3/19
$\boxed{5.}$ $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

3/19
$\boxed{6.}$ $\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$

7. $\dfrac{1-\cos x}{\sin x} + \dfrac{\sin x}{1-\cos x} = 2\csc x$

8. $\dfrac{\sec x - 1}{\sec x + 1} = \dfrac{1-\cos x}{1+\cos x}$

3/20
$\boxed{9.}$ $1 + \cot^2 x = \csc^2 x$

10. $\dfrac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$

3/20
$\boxed{11.}$ $\dfrac{\cot x - 1}{\cot x + 1} = \dfrac{1-\tan x}{1+\tan x}$

12. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

13. $\dfrac{\sin^3 x + \cos^3 x}{\sin x + \cos x} = 1 - \sin x \cos x$

14. $\dfrac{\cos x + 1}{\sin^3 x} = \dfrac{\csc x}{1-\cos x}$

15. $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 4\tan x \sec x$

3/20
$\boxed{16.}$ $\csc^4 x - \cot^4 x = \csc^2 x + \cot^2 x$

17. $\dfrac{\sin^2 x}{\cos^2 x + 3\cos x + 2} = \dfrac{1-\cos x}{2+\cos x}$

18. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$

3/20
$\boxed{19.}$ $\dfrac{1+\tan x}{1-\tan x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

20. $(\sin x - \tan x)(\cos x - \cot x) = (\sin x - 1)(\cos x - 1)$

☆ Examples are worked out for Sample Problems on next 5 pages.

Trig/Precalc 4th period

Mrs. Bledsoe

Name

To be used 3/23-25/2020 (Days 3-5)

3/23/2020 (Monday) Home Instruction Day#3

Law of Sines pg1

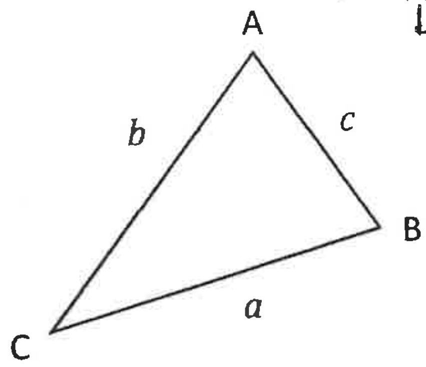
Law of Sines - Examples

For any ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

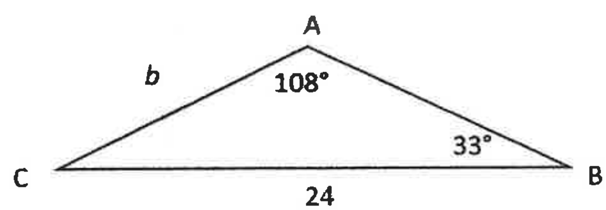


I. Model Problems

In the following example you will find the length of a side of a triangle using Law of Sines.

Example 1:

Find the length of *b*.



Write down known. $a = 24, m\angle A = 108°2, m\angle B = 33°$

Law of Sines $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

Substitute. $\dfrac{24}{\sin 108°} = \dfrac{b}{\sin 33°}$

Simplify. $(\sin 33°)\left(\dfrac{24}{\sin 108°}\right) = b$

Round to the nearest hundredth. $b \approx 13.74$

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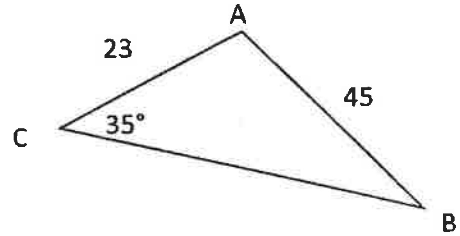
In the following example you will find the measure of an angle of a triangle using Law of Sines.

Monday 3/23/2020 Do#1-8
Tuesday 3/24/2020 DO#9-16
Wednesday 3/25/2020 Do#17-25

Example 2:
Find m∠B.

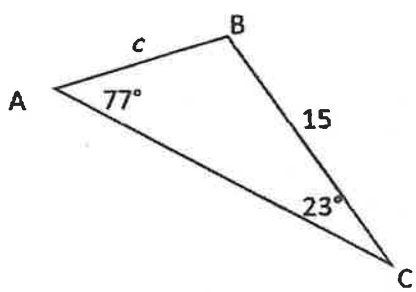
Law of Sines pg2



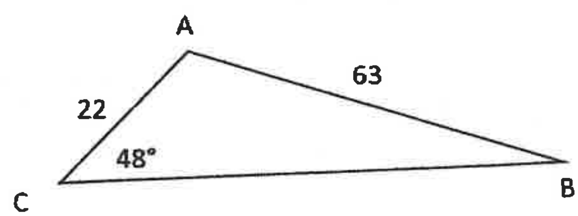
Write down known.	$b = 23, c = 45, m\angle C = 35°$
Law of Sines	$\frac{\sin B}{b} = \frac{\sin C}{c}$
Substitute.	$\frac{\sin B}{23} = \frac{\sin 35°}{45}$
Isolate sin *B*.	$\sin B = \left(\frac{\sin 35°}{45}\right)(23)$
Find the inverse.	$\sin B = \frac{23}{45}(\sin 35°)$
Round to the nearest whole degree.	$m\angle B \approx 17°$

II. Find the length of a side or measure of an angle using Law of Sines. 3/23/2020 Monday #1-8

1. For **ΔABC** find c to the nearest hundredth.



2. For **ΔABC** find m∠*B* to the nearest whole degree.



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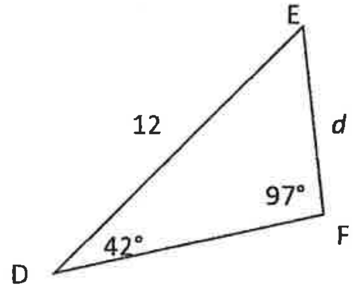
3/23/2020 Cont. #1-8 Home Instruction

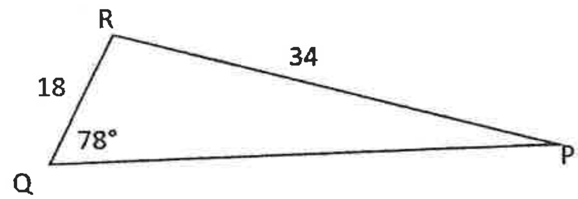
(Monday)

Law of Sines, pg3

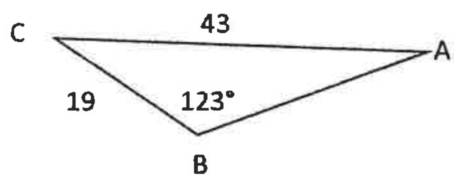
3. For ΔDEF find d to the nearest hundredth.



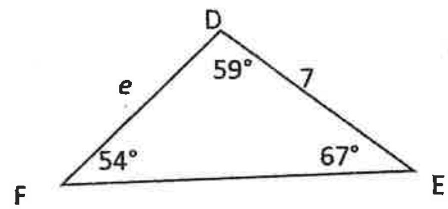
4. For ΔPQR find m∠P to the nearest whole degree.



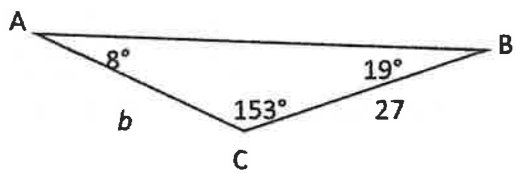
5. For ΔABC find m∠A to the nearest whole degree.



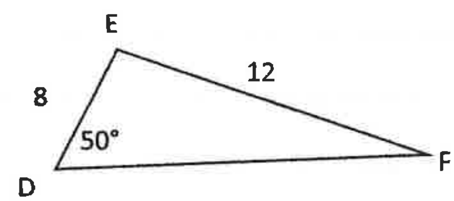
6. For ΔDEF find e to the nearest hundredth.



7. For ΔABC find b to the nearest hundredth.



8. For ΔDEF find m∠F to the nearest whole degree.



End 3/23/2020

#9-16 3/24/2020

Tuesday #9-16

9. For ΔABC, $a = 18$, $b = 6$, and m∠$A = 28°$. Find m∠B to the nearest whole degree.

10. For ΔDEF, $d = 24$, m∠$D = 37°$, and m∠$E = 49°$. Find e to the nearest hundredth.

11. For ΔDEF, $d = 54$, $f = 27$, m∠$D = 20°$. Find m∠F to the nearest whole degree.

12. For ΔABC, $a = 42$, $c = 72$, and m∠$C = 41°$. Find m∠A to the nearest whole degree.

13. For ΔDEF, $e = 34$, m∠$D = 36°$, m∠$E = 72°$, and m∠$F = 72°$. Find d to the nearest whole degree.

14. For ΔXYZ, $x = 17$, m∠$X = 24°$, m∠$Y = 44°$, and m∠$Z = 112°$. Find z to the nearest hundredth

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Tuesday 3/24/2020 #9-16 (then Wed. 3/25/2020 #17-25.)

Law of Sines pg 4

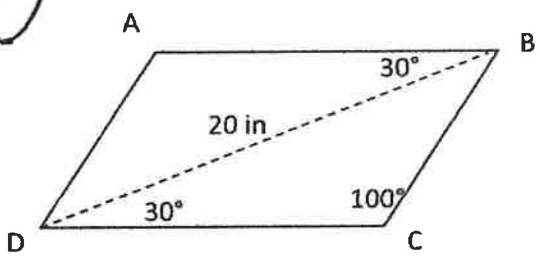
15. For ΔABC, $b = 45, c = 11$, and $m\angle B = 123°$. Find $m\angle C$ to the nearest whole degree.

End 3/24/2020

16. For ΔDEF, $e = 98$, $m\angle D = 52°$, $m\angle E = 71°$, and $m\angle F = 57°$. Find d to the nearest whole degree.

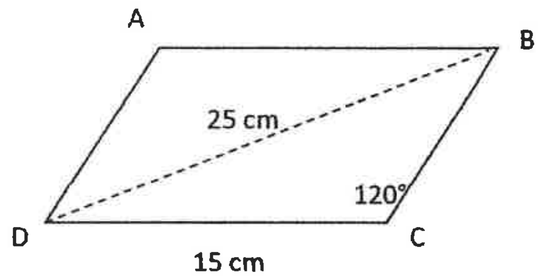
Begin 3/25/2020 #17-25

17. For parallelogram ABCD below find BC to the nearest tenth.



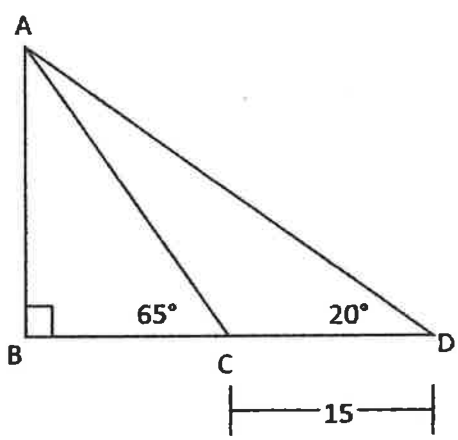
18. For parallelogram ABCD below find $m\angle DBC$ to the nearest whole degree.

Wednesday #17-25

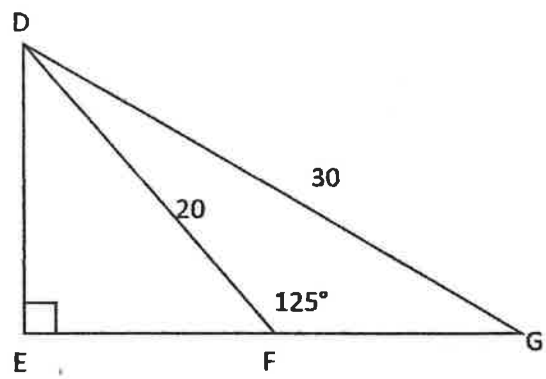


III. Challenge Problems

19. For the figure below find BC to the nearest whole number. **CD=15.**



20. For the figure below find $m\angle EDG$ to the nearest whole degree.

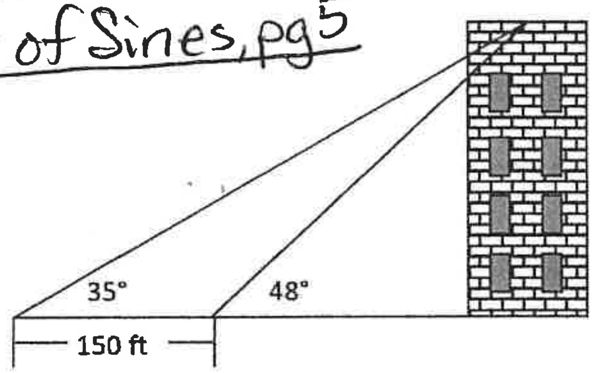


21. Find the height of the building in the figure below to the nearest foot.

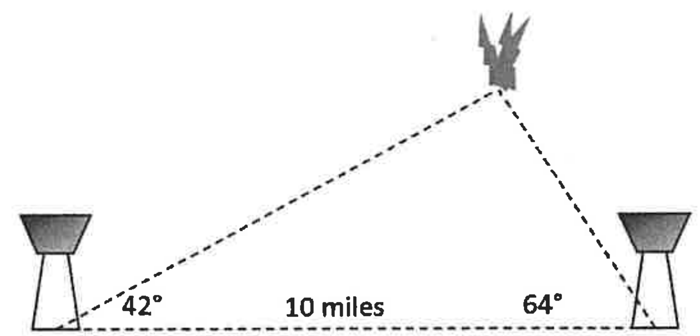
Trig/Precalc 3/25/2020 Continued
Mrs. Bledsoe Home Instruction Name ____________

Law of Sines, pg5

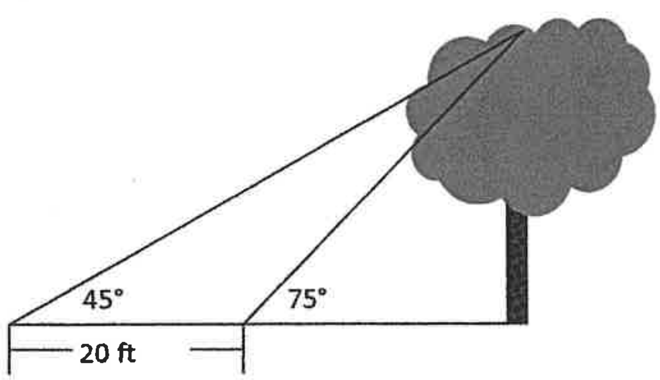
Wednesday
#17-25



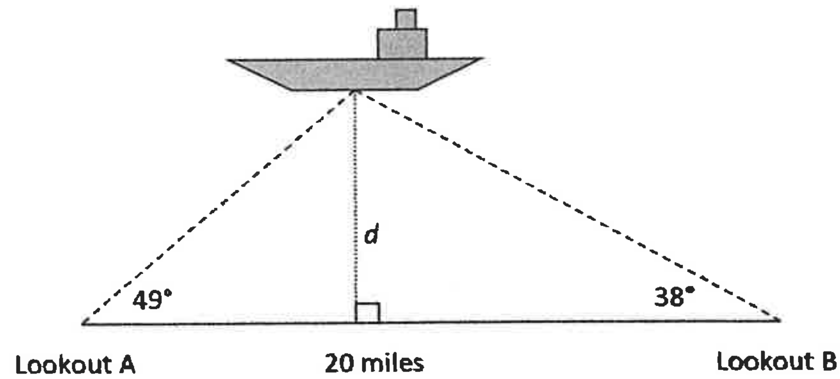
22. Fire towers A and B are located 10 miles apart. They use the direction of the other tower as 0°. Rangers at fire tower A spot a fire at 42°, and rangers at fire tower B spot the same fire at 64°. How far from tower A is the fire to the nearest tenth of a mile?



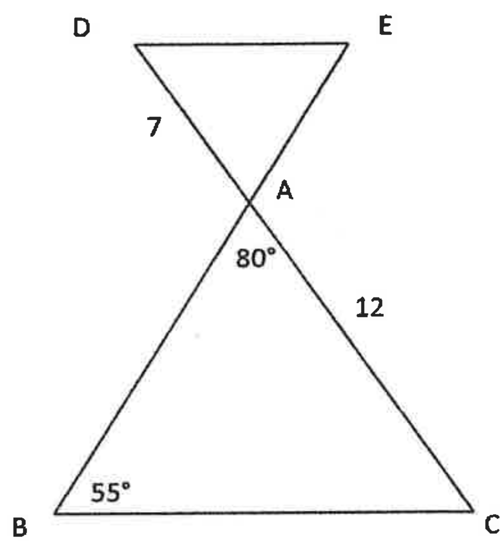
23. Find the height of the tree below to the nearest foot.



24. Triangulation can be used to find the location of an object by measuring the angles to the object from two points at the end of a baseline. Two lookouts 20 miles apart on the coast spot a ship at sea. Using the figure below find the distance, d, the ship is from shore to the nearest tenth of a mile.



25. **ΔDEA~ΔCBA**. Find DE to the nearest whole number.

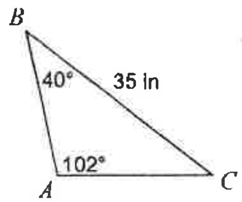


Trigonometry/Precalculus Mrs. Bledsoe Name____________________

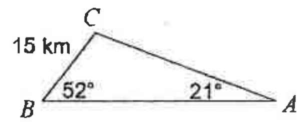
Law of Sines - Home Assignment, Day 6 Date 3/36/2020 (Thursday) 4th Period

Find each side measurement as indicated. Round your answers to the nearest tenth.

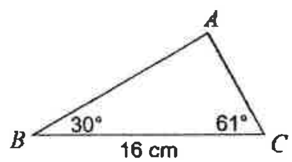
1) Find AC



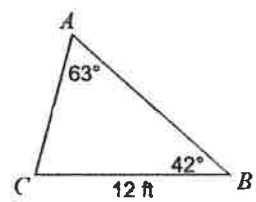
2) Find AC



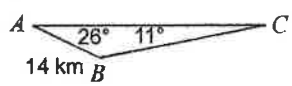
3) Find AB



4) Find AC

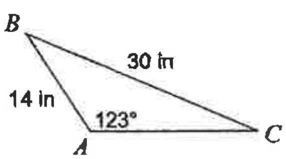


5) Find BC

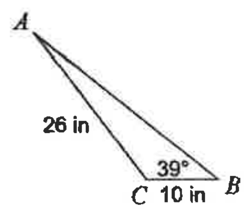


Find each angle as indicated. Round your answers to the nearest tenth.

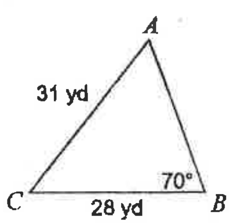
6) Find $m\angle C$



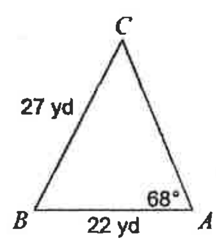
7) Find $m\angle A$



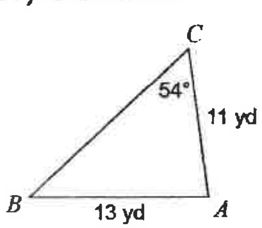
8) Find $m\angle A$



9) Find $m\angle C$



10) Find $m\angle B$

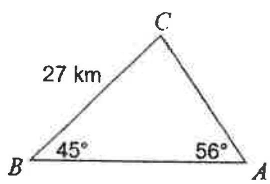


Trigonometry/Precalculus Mrs. Bledsoe Name________________________

Law of Sines - Home Assignment - Day 7 Date 3/27/2020 (Friday) 4th Period

Solve each triangle (Find all missing sides and all missing angles). Round your answers to the nearest tenth.

1)



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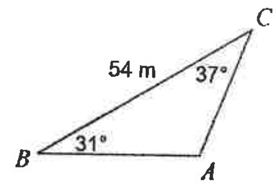
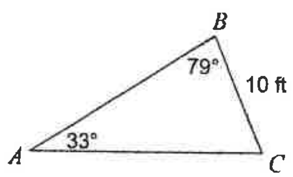


3)



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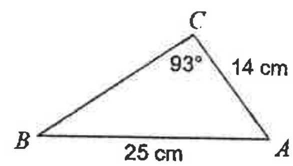

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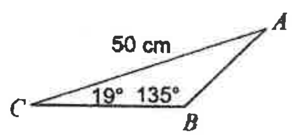

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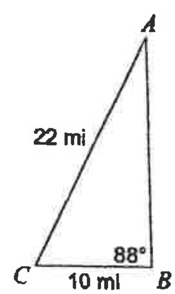
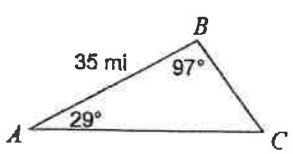


7)



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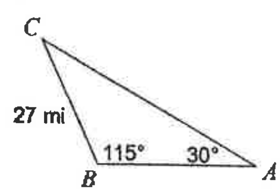


Examples for Trig Identities
(-Does not have to be printed-)

Sample Problems - Examples (3/19-26/2020)

Prove each of the following identities.

1. $\tan x \sin x + \cos x = \sec x$

2. $\dfrac{1}{\tan x} + \tan x = \dfrac{1}{\sin x \cos x}$

3. $\sin x - \sin x \cos^2 x = \sin^3 x$

4. $\dfrac{\cos\alpha}{1+\sin\alpha} + \dfrac{1+\sin\alpha}{\cos\alpha} = 2\sec\alpha$

5. $\dfrac{\cos x}{1-\sin x} - \dfrac{\cos x}{1+\sin x} = 2\tan x$

6. $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$

7. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

8. $\dfrac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$

9. $\dfrac{1-\sin x}{\cos x} = \dfrac{\cos x}{1+\sin x}$

10. $1 - 2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$

11. $\tan^2\theta = \csc^2\theta \tan^2\theta - 1$

12. $\sec x + \tan x = \dfrac{\cos x}{1-\sin x}$

13. $\dfrac{\csc\beta}{\sin\beta} - \dfrac{\cot\beta}{\tan\beta} = 1$

14. $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

15. $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

16. $\dfrac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \dfrac{3+\sin x}{1-\sin x}$

17. $\dfrac{\cos x}{1-\sin x} - \tan x = \sec x$

18. $\tan^2 x + 1 + \tan x \sec x = \dfrac{1+\sin x}{\cos^2 x}$

Examples do not need to be printed.

Sample Problems - Solutions - Examples (3/19-20/2020)

1. $\tan x \sin x + \cos x = \sec x$

Solution: We will only use the fact that $\sin^2 x + \cos^2 x = 1$ for all values of x.

$$\begin{aligned}\text{LHS} &= \tan x \sin x + \cos x = \frac{\sin x}{\cos x} \cdot \sin x + \cos x = \frac{\sin^2 x}{\cos x} + \cos x = \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} \\ &= \frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \text{RHS}\end{aligned}$$

2. $\dfrac{1}{\tan x} + \tan x = \dfrac{1}{\sin x \cos x}$

Solution: We will only use the fact that $\sin^2 x + \cos^2 x = 1$ for all values of x.

$$\text{LHS} = \frac{1}{\tan x} + \tan x = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x} = \text{RHS}$$

3. $\sin x - \sin x \cos^2 x = \sin^3 x$

Solution: We will only use the fact that $\sin^2 x + \cos^2 x = 1$ for all values of x.

$$\text{LHS} = \sin x - \sin x \cos^2 x = \sin x \left(1 - \cos^2 x\right) = \sin x \cdot \sin^2 x = \text{RHS}$$

4. $\dfrac{\cos \alpha}{1 + \sin \alpha} + \dfrac{1 + \sin \alpha}{\cos \alpha} = 2 \sec \alpha$

Solution: We will only use the fact that $\sin^2 x + \cos^2 x = 1$ for all values of x.

$$\begin{aligned}\text{LHS} &= \frac{\cos \alpha}{1 + \sin \alpha} + \frac{1 + \sin \alpha}{\cos \alpha} = \frac{\cos^2 \alpha}{(1 + \sin \alpha)\cos \alpha} + \frac{(1 + \sin \alpha)^2}{(1 + \sin \alpha)\cos \alpha} = \frac{\cos^2 \alpha + (1 + \sin \alpha)^2}{(1 + \sin \alpha)\cos \alpha} \\ &= \frac{\cos^2 \alpha + 1 + 2\sin \alpha + \sin^2 \alpha}{(1 + \sin \alpha)\cos \alpha} = \frac{\cos^2 \alpha + \sin^2 \alpha + 1 + 2\sin \alpha}{(1 + \sin \alpha)\cos \alpha} = \frac{2 + 2\sin \alpha}{(1 + \sin \alpha)\cos \alpha} \\ &= \frac{2(1 + \sin \alpha)}{(1 + \sin \alpha)\cos \alpha} = \frac{2}{\cos \alpha} = 2 \cdot \frac{1}{\cos \alpha} = 2 \sec \alpha = \text{RHS}\end{aligned}$$

5. $\dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x} = 2 \tan x$

Solution: We will start with the left-hand side. First we bring the fractions to the common denominator. Recall that $\sin^2 x + \cos^2 x = 1$ for all values of x.

$$\begin{aligned}\text{LHS} &= \frac{\cos x}{1 - \sin x} - \frac{\cos x}{1 + \sin x} = \frac{\cos x (1 + \sin x)}{(1 - \sin x)(1 + \sin x)} - \frac{\cos x (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{\cos x (1 + \sin x) - \cos x (1 - \sin x)}{(1 - \sin x)(1 + \sin x)} = \frac{\cos x + \cos x \sin x - \cos x + \cos x \sin x}{1 - \sin^2 x} = \frac{2 \sin x \cos x}{\cos^2 x} \\ &= \frac{2 \sin x}{\cos x} = 2 \tan x = \text{RHS}\end{aligned}$$

Examples do not have to be printed. Sample Problems - Examples (3/19-20/2020)

6. $\cos^2 x = \dfrac{\csc x \cos x}{\tan x + \cot x}$

Solution: We will start with the right-hand side. We will re-write everything in terms of $\sin x$ and $\cos x$ and simplify. We will again run into the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$.

$$\begin{aligned}
\text{RHS} &= \frac{\csc x \cos x}{\tan x + \cot x} = \frac{\frac{1}{\sin x} \cdot \cos x}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}} = \frac{\frac{1}{\sin x} \cdot \frac{\cos x}{1}}{\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x}} = \frac{\frac{\cos x}{\sin x}}{\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}} = \frac{\frac{\cos x}{\sin x}}{\frac{1}{\sin x \cos x}} \\
&= \frac{\cos x}{\sin x} \cdot \frac{\cos x \sin x}{1} = \frac{\cos^2 x}{1} = \cos^2 x = \text{LHS}
\end{aligned}$$

7. $\dfrac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = 1$

Solution: We can factor the numerator via the difference of squares theorem.

$$\begin{aligned}
\text{LHS} &= \frac{\sin^4 x - \cos^4 x}{\sin^2 x - \cos^2 x} = \frac{\left(\sin^2 x\right)^2 - \left(\cos^2 x\right)^2}{\sin^2 x - \cos^2 x} = \frac{\left(\sin^2 x + \cos^2 x\right)\left(\sin^2 x - \cos^2 x\right)}{\sin^2 x - \cos^2 x} \\
&= \sin^2 x + \cos^2 x = 1 = \text{RHS}
\end{aligned}$$

8. $\dfrac{\tan^2 x}{\tan^2 x + 1} = \sin^2 x$

Solution:

$$\begin{aligned}
\text{LHS} &= \frac{\tan^2 x}{\tan^2 x + 1} = \frac{\left(\frac{\sin x}{\cos x}\right)^2}{\left(\frac{\sin x}{\cos x}\right)^2 + 1} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x} + 1} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}} \\
&= \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\sin^2 x + \cos^2 x}{\cos^2 x}} = \frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \frac{\sin^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{1} = \sin^2 x = \text{RHS}
\end{aligned}$$

9. $\dfrac{1 - \sin x}{\cos x} = \dfrac{\cos x}{1 + \sin x}$

Solution:

$$\begin{aligned}
\text{LHS} &= \frac{1 - \sin x}{\cos x} = \frac{1 - \sin x}{\cos x} \cdot 1 = \frac{1 - \sin x}{\cos x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{(1 - \sin x)(1 + \sin x)}{\cos x (1 + \sin x)} = \frac{1 - \sin^2 x}{\cos x (1 + \sin x)} \\
&= \frac{\cos^2 x}{\cos x (1 + \sin x)} = \frac{\cos x}{1 + \sin x} = \text{RHS}
\end{aligned}$$

Examples do not have to be printed. Sample Problems – Examples (3/19-20/2020)

10. $1-2\cos^2 x = \dfrac{\tan^2 x - 1}{\tan^2 x + 1}$

Solution:

$$\begin{aligned}
\text{RHS} &= \frac{\tan^2 x - 1}{\tan^2 x + 1} = \frac{\dfrac{\sin^2 x}{\cos^2 x} - 1}{\dfrac{\sin^2 x}{\cos^2 x} + 1} = \frac{\dfrac{\sin^2 x}{\cos^2 x} - \dfrac{\cos^2 x}{\cos^2 x}}{\dfrac{\sin^2 x}{\cos^2 x} + \dfrac{\cos^2 x}{\cos^2 x}} = \frac{\dfrac{\sin^2 x - \cos^2 x}{\cos^2 x}}{\dfrac{\sin^2 x + \cos^2 x}{\cos^2 x}} \\
&= \frac{\sin^2 x - \cos^2 x}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x} = \frac{\sin^2 x - \cos^2 x}{1} = \sin^2 x - \cos^2 x \\
&= \left(1-\cos^2 x\right) - \cos^2 x = 1 - 2\cos^2 x = \text{LHS}
\end{aligned}$$

11. $\tan^2\theta = \csc^2\theta \tan^2\theta - 1$

$$\begin{aligned}
\text{RHS} &= \csc^2\theta\tan^2\theta - 1 = \frac{1}{\sin^2\theta}\cdot\left(\frac{\sin\theta}{\cos\theta}\right)^2 - 1 = \frac{1}{\sin^2\theta}\cdot\frac{\sin^2\theta}{\cos^2\theta} - 1 = \frac{1}{\cos^2\theta} - 1 \\
&= \frac{1}{\cos^2\theta} - \frac{\cos^2\theta}{\cos^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta} = \frac{\sin^2\theta}{\cos^2\theta} = \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \tan^2\theta = \text{LHS}
\end{aligned}$$

12. $\sec x + \tan x = \dfrac{\cos x}{1-\sin x}$

Solution:

$$\begin{aligned}
\text{RHS} &= \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x}\cdot 1 = \frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{(1-\sin x)(1+\sin x)} \\
&= \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \text{LHS}
\end{aligned}$$

13. $\dfrac{\csc\beta}{\sin\beta} - \dfrac{\cot\beta}{\tan\beta} = 1$

Solution: We will start with the left-hand side. We will re-write everything in terms of $\sin\beta$ and $\cos\beta$ and simplify. We will again run into the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$ for all angles x.

$$\begin{aligned}
\text{LHS} &= \frac{\csc\beta}{\sin\beta} - \frac{\cot\beta}{\tan\beta} = \frac{\dfrac{1}{\sin\beta}}{\dfrac{\sin\beta}{1}} - \frac{\dfrac{\cos\beta}{\sin\beta}}{\dfrac{\sin\beta}{\cos\beta}} = \frac{1}{\sin\beta}\cdot\frac{1}{\sin\beta} - \frac{\cos\beta}{\sin\beta}\cdot\frac{\cos\beta}{\sin\beta} = \frac{1}{\sin^2\beta} - \frac{\cos^2\beta}{\sin^2\beta} \\
&= \frac{1-\cos^2\beta}{\sin^2\beta} = \frac{\left(\sin^2\beta + \cos^2\beta\right) - \cos^2\beta}{\sin^2\beta} = \frac{\sin^2\beta}{\sin^2\beta} = 1 = \text{RHS}
\end{aligned}$$

14. $\sin^4 x - \cos^4 x = 1 - 2\cos^2 x$

Solution:

$$\begin{aligned}
\text{LHS} &= \sin^4 x - \cos^4 x = \left(\sin^2 x\right)^2 - \left(\cos^2 x\right)^2 = \left(\sin^2 x + \cos^2 x\right)\left(\sin^2 x - \cos^2 x\right) \\
&= 1\cdot\left(\sin^2 x - \cos^2 x\right) = \left(1-\cos^2 x\right) - \cos^2 x = 1 - 2\cos^2 x = \text{RHS}
\end{aligned}$$

Examples do not have to be printed.

Sample Problems - Examples 3/19-20/2020

15. $(\sin x - \cos x)^2 + (\sin x + \cos x)^2 = 2$

Solution:

$$\begin{aligned} \text{LHS} &= (\sin x - \cos x)^2 + (\sin x + \cos x)^2 \\ &= \left(\sin^2 x + \cos^2 x - 2\sin x \cos x\right) + \left(\sin^2 x + \cos^2 x + 2\sin x \cos x\right) = 2\sin^2 x + 2\cos^2 x \\ &= 2\left(\sin^2 x + \cos^2 x\right) = 2 \cdot 1 = 2 = \text{RHS} \end{aligned}$$

16. $\dfrac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \dfrac{3 + \sin x}{1 - \sin x}$

Solution:

$$\text{LHS} = \frac{\sin^2 x + 4\sin x + 3}{\cos^2 x} = \frac{(\sin x + 1)(\sin x + 3)}{1 - \sin^2 x} = \frac{(\sin x + 1)(\sin x + 3)}{(1 + \sin x)(1 - \sin x)} = \frac{\sin x + 3}{1 - \sin x} = \text{RHS}$$

17. $\dfrac{\cos x}{1 - \sin x} - \tan x = \sec x$

Solution:

$$\begin{aligned} \text{LHS} &= \frac{\cos x}{1 - \sin x} - \tan x = \frac{\cos x}{1 - \sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin x(1 - \sin x)}{\cos x(1 - \sin x)} = \frac{\cos^2 x - \sin x + \sin^2 x}{\cos x(1 - \sin x)} \\ &= \frac{\left(\cos^2 x + \sin^2 x\right) - \sin x}{\cos x(1 - \sin x)} = \frac{1 - \sin x}{\cos x(1 - \sin x)} = \frac{1}{\cos x} = \text{RHS} \end{aligned}$$

18. $\tan^2 x + 1 + \tan x \sec x = \dfrac{1 + \sin x}{\cos^2 x}$

Solution:

$$\begin{aligned} \text{LHS} &= \tan^2 x + 1 + \tan x \sec x = \frac{\sin^2 x}{\cos^2 x} + 1 + \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} + \frac{\sin x}{\cos^2 x} \\ &= \frac{\sin^2 x + \cos^2 x + \sin x}{\cos^2 x} = \frac{1 + \sin x}{\cos^2 x} = \text{RHS} \end{aligned}$$

For more documents like this, visit our page at http://www.teaching.martahidegkuti.com and click on Lecture Notes. E-mail questions or comments to mhidegkuti@ccc.edu.

 Last revised: May 8, 2013

Mrs. Bledsoe 4th Period Trig/Precalc

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas (alternate form)

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Inverse Trig Functions

Definition

$y=\sin^{-1} x$ is equivalent to $x=\sin y$

$y=\cos^{-1} x$ is equivalent to $x=\cos y$

$y=\tan^{-1} x$ is equivalent to $x=\tan y$

Domain and Range

Function	Domain	Range
$y=\sin^{-1} x$	$-1\le x\le 1$	$-\dfrac{\pi}{2}\le y\le \dfrac{\pi}{2}$
$y=\cos^{-1} x$	$-1\le x\le 1$	$0\le y\le \pi$
$y=\tan^{-1} x$	$-\infty< x< \infty$	$-\dfrac{\pi}{2}< y< \dfrac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right)=x \qquad \cos^{-1}\left(\cos(\theta)\right)=\theta$

$\sin\left(\sin^{-1}(x)\right)=x \qquad \sin^{-1}\left(\sin(\theta)\right)=\theta$

$\tan\left(\tan^{-1}(x)\right)=x \qquad \tan^{-1}\left(\tan(\theta)\right)=\theta$

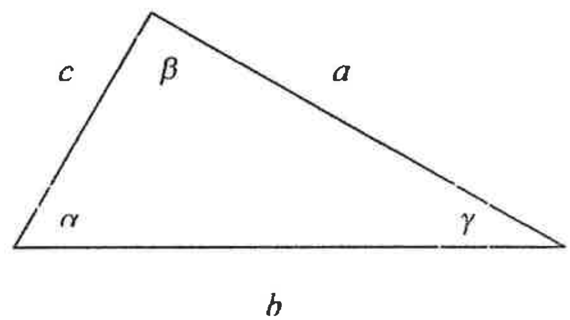
Alternate Notation

$\sin^{-1} x=\arcsin x$

$\cos^{-1} x=\arccos x$

$\tan^{-1} x=\arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2=b^2+c^2-2bc\cos\alpha$$

$$b^2=a^2+c^2-2ac\cos\beta$$

$$c^2=a^2+b^2-2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c}=\frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

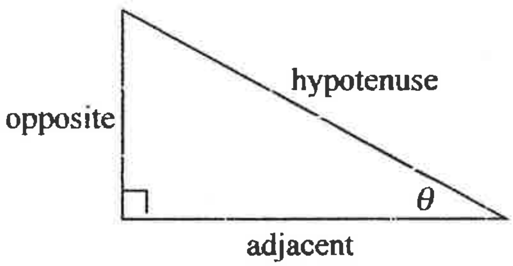
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



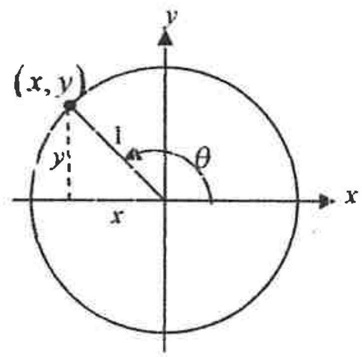
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition

For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$

$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$

$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

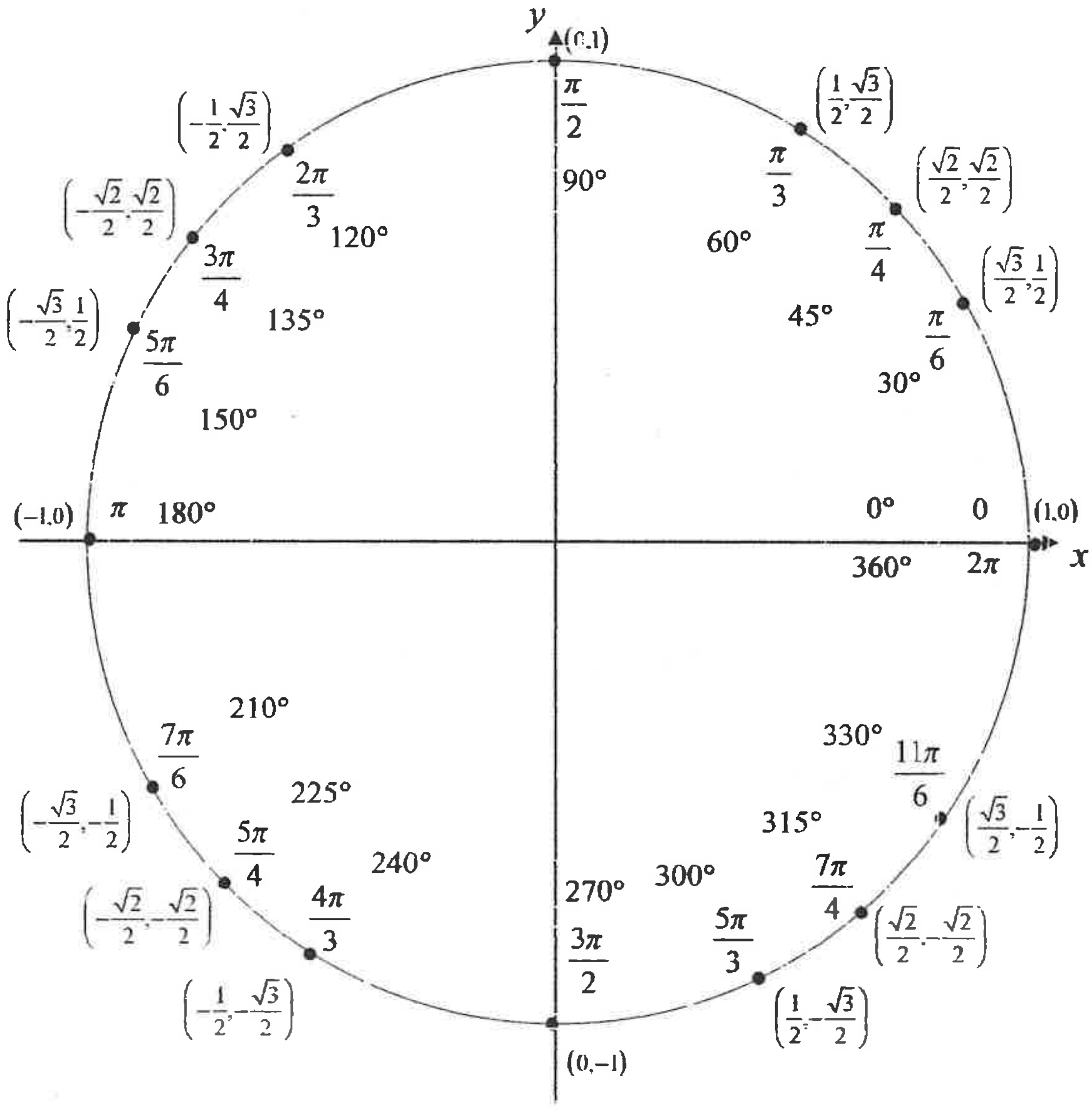
$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$